Parameter choice rules under qualitative noise conditions

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The choice of the correct regularization parameter is the most important part when computing a regularization for ill-posed problems. It strongly determines the quality of the reconstructed approximate solution and optimal convergence is only obtained by an appropriate selection of the regularization parameter. Moreover, a large portion of the theory of the regularization of ill-posed problems is concerned with proving convergence and convergence rates using certain parameter choice rules.

Roughly one can distinguish between different types of parameter choice rules, depending on what information is used in the parameter choice rules (PCR): a) PCRs which uses information on the exact solution, b) PCRs which use information on the noise level, and c) PCRs which use neither information on the noise levels nor on the exact solution. While the cases a), and b) are rather well established, we would like to focus on the case c), which associated PCRs are sometimes called noiselevel-free (or heuristic) parameter choice rules.

From a practical point of view, noiselevel-free parameter choice rules are the most useful one, however, they suffer from a serious drawback: the Bakushinskii veto. It is a well-known result of Bakushinskii that a parameter choice rule, that uses only the data and no information on the noiselevel and the solution cannot converge in the worst case.

However, some recent result showed that the noiselevel-free parameter choice rules can be saved if a restricted noise case is assumed: Under certain noise conditions, several noiselevel-free parameter choice rules show convergence as the noise-level vanishes. In this sense, PCRs of case c) can be viewed as convergent rules, which implicitly use a qualitative restriction of the noise (instead of a quantitative as the noise level).

In this talk we would like to discuss the main results in this field, such as the qualitative restrictions on the noise and possibly on the exact solution and the convergence and convergence rates result for several well-known noiselevel-free PCRs in the continuous and discrete setting.

References:

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