ABSTRACTS

of the 10th International Conference "Inverse Problems: Modeling & Simulation"

May 22-28, 2022, Paradise Bay Resort Hotel, Malta

Editors
Alemdar Hasanov Hasanoglu, Roman Novikov, Otmar Scherzer,
Cristiana Sebu and Eric Todd Quinto

Managing Editor
Burhan Pektaş

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THE EURASIAN ASSOCIATION ON INVERSE PROBLEMS

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The Conference is organized by The Eurasian Association on Inverse Problems and supported by the international journals Inverse Problems and Journal of Inverse and Ill-Posed Problems.

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Preface

The Tenth International Conference "Inverse Problems: Modeling and Simulation" is held during May 22 – 28, 2022, in the Paradise Bay Resort Hotel, Malta, under the auspices of The Eurasian Association on Inverse Problems (EAIP), and also the leading international journals Inverse Problems and Journal of Inverse and Ill-Posed Problems.

As with the previous nine conferences in this series, the main objective of this conference is to be multidisciplinary and international, by bringing together scientists working on various topics of inverse problems in diverse areas, such as mathematics, statistics, engineering, economics, finance, physics, chemistry, biology, medicine, meteorology and computer science.

The conference brings together more than 200 internationally known experts on inverse problems, and exhibitors from over 30 countries world-wide.

The conference program includes 6 plenary lectures as well as invited lectures given in the framework of 28 minisymposiums. The topics of the conference go through advances in inverse and ill-posed problems over the past 20 years as well as emerging methods in data science.

We hope that this book of abstracts will be useful to those who are interested in inverse and ill-posed problems: theory, numerical implementations and applications.

We would like to thank Mr. Kenan Kocatürk, founder and manager of the publishing house Literatür, for his support.

PLENARY SPEAKERS

QUANTITATIVE INVERSE SCATTERING VIA REDUCED ORDER MODELING

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Abstract

We will discuss an inverse problem for the wave equation, where an array of sensors probes an unknown heterogeneous medium with waves and measures the echoes. The goal is to determine scattering structures in the medium modeled by a reflectivity function.

Much of the existing imaging methodology is based on a linear least squares data fit approach. However, the mapping between the reflectivity and the wave measured at the array is nonlinear and the resulting images have artifacts. We will show how to use a reduced order model (ROM) approach to solve the inverse scattering problem. The ROM is data driven i.e., it is constructed from the data, with no knowledge of the medium. It approximates the wave propagator, which is the operator that maps the wave from one-time step to the next. We will show how to use the ROM to:

(1) Remove the multiple scattering (nonlinear) effects from the data, which can then be used with any linearized inversion algorithm.

(2) Obtain a well-conditioned quantitative inversion algorithm for estimating the reflectivity

ILL-POSEDNESS CONCEPTS AND DISTINGUISHED ROLE OF SMOOTHNESS IN REGULARIZATION FOR LINEAR AND NONLINEAR INVERSE PROBLEMS

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Abstract

In a first part, we consider different concepts of ill-posedness for mathematical models of inverse problems expressed by linear and non-linear operator equations in infinite dimensional Hilbert and Banach spaces. The concepts of Hadamard and Nashed are recalled, which are appropriate for linear operator equations. Compact linear forward operators in Hilbert spaces allow for a pre-selection of the degree of ill-posedness by verifying the decay rate of the singular values indicating the operator smoothness. The interplay of operator smoothness and solution smoothness is responsible for opportunities and limitations of stable approximate solutions to inverse problems by regularization. For non-linear operator equations, the nature of ill-posedness may vary with the solution point. The presented concept of local ill-posedness is adapted to that feature.

In a second part, the impact of different varieties of smoothness on variants of variational regularization is under consideration. In this context, variational source conditions represent a sophisticated tool for expressing the solution smoothness with respect to the character of the forward operator, for non-linear problems even in combination with the occurring structure of non-linearity. Also the role of conditional stability estimates in combination with Tikhonov regularization is outlined. Several example situations for applying regularization approaches are presented, including sparsity-promoting versions. We also discuss the specific difficulties and some recent results for the Tikhonov regularization with oversmoothing penalties.

This research is supported by the German Research Foundation (DFG) under grant HO 1454/12-1 and embedded in the Austrian/German joint research project "Novel Error Measures and Source Conditions of Regularization Methods for Inverse Problems (SCIP)" with Prof. Otmar Scherzer (Vienna) based on the D-A-CH Lead-Agency Agreement.

ACOUSTICALLY MODULATED ELECTROMAGNETIC INVERSE SOURCE PROBLEMS

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Abstract

The inverse source problem for the Maxwell equations is of fundamental interest and considerable practical importance, with applications ranging from geophysics to biomedical imaging. The problem is usually stated in the following form: determine the electric current density from boundary measurements of the electric and magnetic fields. It is well known that this problem is underdetermined and does not admit a unique solution.

In this work we propose an alternative approach to the electromagnetic inverse source problem. In this approach the electric current density as well as the conductivity, electric permittivity and magnetic permeability are spatially modulated by an acoustic wave. In this manner, we find that it is possible to uniquely recover the current density from boundary measurements of the fields with Lipschitz stability.

Numerical simulations are used to illustrate the analytical results.

HYPERMODELS, SPARSITY AND APPROXIMATE BAYESIAN COMPUTING

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Abstract

In numerous applications involving an underdetermined large scale inverse problem, sparsity of the solution is a desired property. In the Bayesian framework of inverse problems, sparsity requirement of the solution may be implemented by properly defining the prior distribution. While Gaussian priors are not well suited for promoting sparsity, certain hierarchical, conditionally Gaussian models have been demonstrated to be efficient. In this talk, we review a general class of conditionally Gaussian hypermodels that provide a flexible framework for promoting sparsity of the solution, and discuss approximate iterative methods that can be used both for finding single sparse solutions as well as for approximate sampling of the posterior distributions.

The motivation for this work comes from an ongoing work on brain imaging using magnetoencephalography data.

INVERSE PROBLEMS FOR NONLINEAR EQUATONS

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Abstract

We will survey recent developments in the solution of inverse problems for nonlinear equations. The nonlinearity helps to solve several inverse problems that cannot be solved for the linearized equations.

DIRECT SAMPLING-TYPE METHODS FOR GENERAL NONLINEAR ILL-POSED INVERSE PROBLEMS

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Abstract

In this talk we will have an overall review of the systematic developments of direct sampling methods (DSMs) for solving general nonlinear inverse problems of PDEs. DSMs were initially proposed for solving inverse acoustic scattering problems with either far-field or near-field data [7][8], then developed for inverse electromagnetic scattering problems [6], and further extended for solving several representative non-wave type inverse problems, including electric impedance tomography[5], diffusive optical tomography [3], inversion of Radon transform [1], as well as recovering moving inhomogeneous inclusions [4]. DSMs have also been proved recently to be applicable to the simultaneous reconstruction of inhomogeneous inclusions of different physical nature [2]. The DSMs are computationally cheap, highly parallel, and robust against noise, particularly applicable to the cases when very limited data is available. General motivations, principles and justifications of DSMs are presented in this talk. Numerical experiments are demonstrated for various inverse problems.

There are quite intensive studies of DSMs, and some of the references for those most representative inverse problems are listed below.

These research projects were supported by Hong Kong RGC General Research Fund (Projects 14306921 and 14306719).

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MINISYMPOSIUMS

MINISYMPOSIUM

M1: Recent Advances in Inverse Problems over the Past 20 Years

Commemorative Minisymposium for the 10th IPMS Conference

Organizers:
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Eric Todd Quinto, Tufts University, USA, Todd.Quinto@tufts.edu

The First IPMS Conference was organized by John R. Cannon, Alemdar Hasanov, Zuhair M. Nashed, Albert Tarantola, Alex Tolstoy and Fadil Santosa, in 2002, in Öludeniz - Fethiye, Turkey. Since then, these meetings have occurred biennially. The following traditional features of the conference have been important aspects of the conferences from the beginning: friendship, collaboration and support of young scientists; celebrating the anniversaries of our outstanding colleagues; rewarding young scientists in order to encourage them; rewarding our distinguished colleagues The EAIP (Eurasian Association on Inverse Problems) Award to recognize their outstanding scientific contributions to the field of inverse problems and continuous efforts to foster cooperation between researchers of Eurasian countries. Interest in the IPMS conferences has been steadily growing over the last decade. Every effort has been made to maintain those traditional features of IPMS Conferences, and this has made these conferences unique and exciting, starting with the inaugural meeting in 2002.

This minisymposium consists of topical survey lectures by distinguished experts on inverse problems who have played an extraordinary role in establishing the tradition of the IPMS conference series as well as lectures from EAIP Award and EAIP Young Scientist Award winners.

We extend our thanks to all our excellent keynote and minisymposium speakers, conference attendees, students, exhibitors and guests who have made these IPMS conferences most successful and exciting events.

UNIQUENESS AND STABILITY QUESTIONS FOR INVERSE RADON TRANSFORMS

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Abstract

In Computerized Tomography it is often desirable to reconstruct a function $f(x)$ on a proper subset S of the support of f, the region of interest, from measurements of the Radon transform $Rf(L)$ on the set of lines L that intersect S. Well known examples show that this is not possible in general. Instead it was believed for a while that (in dimension 2) it was necessary to know the full Radon transform Rf for computing f in a region of interest. However, beginning around 2000 new methods were invented that made it possible in some cases to reconstruct f in a region of interest based on considerably less than complete data, [1]. For instance, if J is a segment of a line L_0 that contains $L_0 \cap$ supp f, and $Rf(L)$ is known for all lines L that intersect J, then one can compute the (one-dimensional) Hilbert transform of the restriction of f to J. This has led to much recent work on the Region of Interest problem, for instance to study of uniqueness and stability for inverting the so-called truncated Hilbert transform.

In SPECT one measures the weighted Radon transform

$$\rho(\omega, x) = \exp\left(- \int_0^\infty a(x + t\omega^{\perp}) dt \right)$$

Already in the 1980s inversion of R_ρ was performed in hospitals, although it was not known if R_ρ was injective on compactly supported f for arbitrary attenuation function $a(x)$. In 2002 Roman Novikov solved this problem by giving an explicit formula for the solution f of the equation $R_\rho f = g$, [2].

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INVERSE PROBLEMS FOR A VIBRATNG EULER-BERNOULLI BEAM BASED ON BOUNDARY MEASURED OUTPUTS

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Abstract

In this talk, we will describe inverse problems based on novel mathematical models emerging from important engineering applications. Based on unified approach, we will analyze the distincyive properties of the following three types of inverse problems: inverse source problems, inverse boundary value problems and inverse coefficient problems formulated for the variable coefficients dynamic Euler-Bernoulli equation

$$m(x)u_{tt} + \mu(x)u_{tt} + r(x)u_{xx} = F(x,t) \;\; (x,t) \in \Omega_T := \left\{(x,t) \in R^2 : x \in (0,l), t \in (0,T)\right\}.$$

We will outline reconstruction methods and some benchmark examples to illustrate our results.

These inverse problems are based on the following types available boundary measured outputs: measured output: the measured deflection, slope or moment [1-5]. The proposed approach is based on a weak solution theory for PDEs, Tikhonov regularization combined with the adjoint method [5]. The Neumann-to-Dirichlet, as well as Neumann-to-Neumann operators corresponding to the inverse coefficient problems, are introduced. It is shown that these operators are compact and Lipschitz continuous. The last property allows us to prove the existence of a quasi-solution of the considered inverse problems. Frechet dierentiability of the corresponding Tikhonov functionals are proved and explicit formulas for the Frechet gradients are derived by making use of the unique solutions of the corresponding adjoint problems.

Numerical examples with random noisy measured outputs are presented to illustrate the validity and effectiveness of the proposed approach.

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MULTIDIMENSIONAL INVERSE SCATTERING PROBLEM

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Abstract

We give a short review of old and recent results on the multidimensional inverse scattering problem for the Schrödinger equation. A special attention is paid to approximate but mathematically justified and numerically efficient reconstructions of the potential from scattering data which can be measured in practice.

In particular, our considerations include reconstructions from monochromatic scattering data which are nonoverdetermined or/and phaseless. Potential applications include acoustic tomography, tomographies using elementary particles and phaseless inverse X-ray scattering.

This talk is based, in particular, on the works [1- 5]

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INVERSE PROBLEMS OF SINGLE MOLECULE LOCALIZATION MICROSCOPY

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Abstract

Single molecule localization microscopy (SMLM) is a high resolution imaging technique, which allows to visualize single molecules. The basic principle consists in chemically attaching fluorescent dyes to the molecules, which after excitation with a strong laser light emit light. In this paper we derive some models describing the propagation of light in SMLM experiments via Maxwell's equations. This is the basis of formulating inverse problems related to SMLM. We show that the current status of practical reconstruction can be derived as a simplified solution of the general inverse problem.

This is joint work with Montse Lopez Martinez (TU Vienna), Kamran Sadiq (RICAM, Linz), Magdalena Schneider (TU Vienna), John Schotland (University of Michigan), Gerhard Schuetz (TU Vienna).

TOPOLOGICAL DERIVATIVE BASED METHODS FOR SHAPE RECONSTRUCTION

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Abstract

Solving inverse problems related with the detection of multiple objects and the reconstrucion of their shapes when no a priori information about their number, size or location is provided, is an important field in Applied Mathematics and Physics that arises in a large number of different industrial and engineering applications such as non-destructive testing, geophysical exploration, biomedicine, radar imaging and antenna design.

In this work we propose numerical methods to solve inverse obstacle problems by topological derivative based methods. The topological derivative of a shape functional measures the sensitivity of such functional to having an infinitesimal object at each point of the region of interest. The idea behind these methods is to generate an indicator function able to classify each point as either belonging to the background medium or to an object, without any a priori assumption. We will show closed-form formulae of this derivative for several model problems, which will involve the solution to a forward problem and to a related adjoint one.

The performance of these methods will be illustrated throughout a wide gallery of numerical experiments covering applications that include inverse scattering problems in 2D and 3D acoustics [1,2,3], electromagnetism [4,5], holographic 3D imaging [6] and inverse heat conduction problems [7].

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A SIMPLE EXAMPLE OF HÖLDER-LOGARITHMIC STABILITY IN INVERSE PROBLEMS

Michail Isaev[a] and Roman G. Novikov[b]
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Abstract

Many important inverse problems are exponentially ill-posed in general, which constitutes a severe difficulty for numerical treatments. However, a stable reconstruction of the unknown parameter might still be possible in some cases when the parameter is well-behaved. Countless results in the literature confirm improved stability under various additional a-priori information. In fact, the behaviour of the stability bounds changes dramatically from the logarithmic type to the Hölder type or even, under some strong assumptions, to the Lipchitz type. In this talk, we illustrate such transitions with an example from the classical Fourier analysis.

THE IDENTIFICATION OF A TIME-DEPENDENT SOURCE IN A TIME-FRACTIONAL DIFFUSION EQUATION WITH NON-SMOOTH SOLUTIONS

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Abstract

In this talk, an inverse source problem (ISP) for a time-fractional diffusion equation of order $\alpha\in(0,1)$ where the coefficients of the elliptic operator are dependent on spatial and time variables is discussed. The missing solely time-dependent source is recovered from an additional integral measurement. First, the uniqueness of a solution to the ISP will be shown. Next, two numerical algorithms will be proposed based on Rothe's method over uniform and graded grids, and the convergence of iterates towards the exact solution will be discussed. An essential feature of the fractional subdiffusion problem is that the solution lacks smoothness near the initial time, although it would be smooth away from $t = 0$. Rothe's method on a uniform grid addresses the existence of such a solution (non-smooth with $t^\\gamma$ term where $1>\gamma > \alpha$) under low regularity assumptions, whilst Rothe's method over graded grids has the advantage to cope better with the behaviour at $t=0$ (also here $t^\\alpha$ is included in the class of admissible solutions) for the considered problems. Numerical experiments will support these theoretically obtained results.

This talk is based on joint work with A.S. Hendy (Ural Federal University).

PROPERTIES OF THE STEKLOV ZETA FUNCTION OF A SMOOTH PLANAR DOMAIN OVER THE REAL AXIS

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Abstract

We address the question of reconstructing a bounded smooth and simply connected planar domain Ω from the spectrum of its Dirichlet-to-Neumann operator (Steklov spectrum). We state analog formulations of this problem, one of them arising from Electrical Impedance Tomography.

First we review estimates from below for the Steklov spectral zeta function ζ_Ω on the real intervals $(-\infty,-1)$ and $(1,\infty)$ Additional estimates are then given for the values of ζ_Ω at the negative even integers, the so-called zeta invariants introduced in [1]. A compactness theorem for isospectral families of planar domains is derived, which generalizes a previous pre-compactness result given in [2].

Then we introduce a first variation formula for ζ_Ω when one smoothly deformes the domain Ω. We extends previous estimates for ζ_Ω to the interval $(-1,1)$ by choosing an appropriate path of deformation. These results also generalize a result in [3] on the derivative $\zeta'_\Omega(0)$.

We also discuss application of the first variation formula to the question of nonexistence of nontrivial isospectral deformations of a planar domain.

This talk is based on joint works [4,5,6] and works in progress with Vladimir Sharafutdinov (Novosibirsk State University & Sobolev Institute of Mathematics).

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MINISYMPOSIUM
M2: New Trends in Regularization Theory

Organizers:
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Due to the ill-posedness of most linear and nonlinear inverse problems one needs regularization techniques for their stable approximate solution. The theory is well developed for problems in Hilbert spaces and during the last years many results have also been achieved for problems in Banach spaces. However, there are still many challenging questions, and permanently new classes of inverse problems occur, motivated by applications from natural sciences, engineering and finance.

We want to bring together experts and young researches working in this field to discuss about new results in the analysis and numerics of inverse and ill-posed problems.

A MODIFIED ASYMPTOTICAL REGULARIZATOIN OF NONLINEAR ILL-POSED PROBLEMS

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Abstract

In this talk, we investigate the continuous version of modified iterative Runge–Kutta-type methods for nonlinear inverse ill-posed problems. The convergence analysis is proved under the tangential cone condition, a modified discrepancy principle, i.e., the stopping time T is a solution of $\left\| F(x^{\delta}(\tau)) - y^{\tau} \right\| = \tau\delta^{+}$ for some $\delta^{*} > \delta$ and an appropriate source condition.

We yield the optimal rate of convergence.

ON THE DEAUTOCONVOLUTION PROBLEM IN THE TWO-DIMENSIONAL CASE

Yu Deng
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Abstract

In the past years, there have been many contributions to the theory and practice of deautoconvolution as a nonlinear inverse and ill-posed problem, with applications to stochastics, physics and other natural sciences. All these contributions have focused on the one-dimensional case, aimed at the stable approximate reconstruction of real or complex functions over some finite interval based on noisy observations of the autoconvolution of the desired function. The complex case was discussed in [1] for an application in laser optics.

In the present talk, we try to present first ideas and results of numerical case studies concerning the two-dimensional case. This case occurs when images (e.g., pictures over the unit square) have to be reconstructed from corresponding two-dimensional autoconvolution functions.

Reference

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A NEW WAY OF INTERPRETING TIKHONOV REGULARIZATION AND ITS CONSEQUENCE FOR THE ESTIMATION OF SOLUTION SMOOTHNESS AND NOISE LEVEL

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Abstract

In the theory for the regularization of ill-posed inverse problems, two parameters are essential for the description of regularization properties: noise level and a source condition, the latter characterizing the interplay between solution smoothness and smoothing of the forward operator. Unfortunately, in many practical problems neither of the parameters is available. In this talk we present an approach that allows to estimate the solution smoothness in terms of an exponent in a source condition and the noise level. The method is based on how Tikhonov regularization approximates an unknown solution. We show that one can use the established concept of approximate source conditions and generalize them to not only describe the approximation of the unknown, but also the perturbed data and the noise component. Using the relation between the approximation functions, we derive in a new way the well-known results on optimal convergence rates and parameter choice rules but also reveal new relations between them.

The new approach is not restricted to Tikhonov regularization, we demonstrate that Landweber iteration follows the same general principles.

ON REGULARIZED PROJECTION METHODS FOR ILL-POSED PROBLEMS

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Abstract

We consider in Hilbert spaces H, F linear ill-posed problem

$$Au = f, \qquad A \in \mathcal{L}(H, F), \qquad f \in \mathcal{R}(A) \neq \overline{\mathcal{R}(A)}. \tag{1}$$

Only noisy data f^δ satisfying $\|f - f^\delta\| \leq \delta$ are available . Let P_n, Q_n $(n \in \mathbb{N})$ be the orthoprojectors in H, F respectively, satisfying $\|A(I - P_n)\| \to 0$, $\|(I - Q_n)A\| \to 0 \quad (n \to \infty)$. The projection method for (1) has the form $A_n u_n = Q_n f^\delta, \quad A_n = Q_n A P_n, \quad u_n \in \mathcal{R}(P_n)$. We regularize this equation using generating function $g_r : [0, a] \to \mathbb{R} \quad (\|A\| \leq a, r \geq 0)$ with properties

$$|g_r(\lambda)| \leq \gamma r \qquad (0 \leq \lambda \leq a, \quad \gamma = const),$$

$$\lambda^p |1 - \lambda g_r(\lambda)| \leq \gamma_p r^{-p} \qquad (0 \leq \lambda \leq a, \quad 0 \leq p \leq p_0, \quad p_0 > 1/2, \quad \gamma_p = const).$$

Let $u_0 \in H$ and let u_* be the solution of (1), nearest to u_0. We find

$$u_{n,r} = (I - g_r(A_n^* A_n) A_n^* A_n) P_n u_0 + g_r(A_n^* A_n) A_n^* f^\delta.$$

For the choice of r we choose $b_2 \geq b_1 \geq 1$ and use the following discrepancy principle. If $\|A_n u_0 - Q_n f^\delta\| \leq b_1 \delta$, choose $r = 0$. Otherwise define $\overline{r} := \sup_{q>0} [(2q)^{1/q} \|(I - P_n)|A|^q\|^{-2/q}]$ with $|A| = (A^* A)^{1/2}$ and choose $r \in (0, \overline{r})$ such that $b_1 \delta \leq \|A_n u_{n,r} - Q_n f^\delta\| \leq b_2 \delta$. If the last condition is violated for all values $r \leq \overline{r}$, choose $r = \overline{r}$ or $r = \lfloor \overline{r} \rfloor + 1$.

This choice of r guarantees convergence $u_{n,r} \to u_* (\delta \to 0, n \to \infty)$ and under assumptions

$$u_* = |A|^p z, \quad \|z\| \leq \rho, \qquad u_* - u_0 = |A|^p v, \quad \|v\| \leq \rho, \quad p \leq 2p_0 - 1 \tag{2}$$

also the error estimate

$$\|u_{n,r} - u_*\| \leq const[\rho^{\frac{1}{p+1}} \delta^{\frac{p}{p+1}} + \rho \|(I - P_n)|A|^p\| + \rho e_\mu(Q_n)], \tag{3}$$

where

$$e_\mu(Q_n) = \begin{cases} \|(I - Q_n)|A^*|^\mu\|^{\min(\frac{p}{\mu}, 2)} \quad \forall \mu \in (0, 1], \quad \mu \neq p/2 & \text{if } p \leq 2; \\ \|(I - Q_h)A\| \|(I - Q_h)|A^*|^{p-1}\| & \text{if } p \geq 2. \end{cases}$$

Consider regularization of the projected equation by the m times iterated Tikhonov method with corresponding generating function $g_{r,m}(\lambda) = [1 - (\frac{1}{1+r\lambda})^m]/\lambda$, $p_0 = m$, $m \geq 1$. Then for the choice of r instead of the discrepancy principle the monotone error rule (ME-rule) can be recommended, replacing in the rule formulated above the discrepancy $\|A_n u_{n,r,m} - Q_n f^\delta\|$ by the ME-function

$$\frac{(A_n u_{n,r,m} - Q_n f^\delta, A_n u_{n,r,m+1} - Q_n f^\delta)}{\|A_n u_{n,r,m+1} - Q_n f^\delta\|}.$$

Then the error estimate (3) is true for $p \leq 2p_0 = 2m$ (instead of the original $p \leq 2p_0 - 1 = 2m - 1$).

Note that obtained estimates for the discretization error are sharper than in well-known works (cf. [1]).

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STOCHASTIC GRADIENT DESCENT IN BANACH SPACES

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Abstract

Stochastic gradient descent and its many variants are among the most promising and successful approaches for computing iterative solutions to optimization problems. They have outperformed traditional methods in many problem settings and have seen increasing use for inverse problems. In each iteration, stochastic gradient methods use only a small subset of data points to compute the estimator of the gradient. This results in methods that scale well to problem size, which made them a popular choice for an increasing number of problems, and in particular they hold significant promise for solving large-scale inverse problems.

Stochastic gradient methods are still predominantly used only in inner product spaces which provide a range of geometric and analysis tools that facilitate their study. Convergence of SGD for solving linear inverse problems is in these settings well understood [2]. However, the native domain of many inverse problems is in Banach spaces, such as parameter identification in elliptic PDEs, and Banach space norms are advantageous for e.g. preservation of sparsity. Gradient descent-based reconstruction methods for solving linear inverse problems in Banach spaces in the deterministic setting have been previously studied [1], and they typically follow Landweber type iterations

$$x_{k+1} = \mathcal{J}_{q^*}\left(\mathcal{J}_q(x_k) - \alpha_k A^* \mathcal{J}_p(Ax_k - y)\right). \tag{1}$$

Mappings $\mathcal{J}_q : \mathcal{X} \to \mathcal{X}^*$, $\mathcal{J}_{q^*} : \mathcal{X}^* \to \mathcal{X}$, $\mathcal{J}_p : \mathcal{Y} \to \mathcal{Y}^*$ are the so-called duality mappings, with parameters $p, q \geq 1$ and q^* such that $1/q + 1/q^* = 1$. The search direction $A^* \mathcal{J}_p(Ax_k - y)$ is the sub-gradient of the of the p^{th} power of the Banach space norm, lending a gradient descent interpretation of the iterates. Due to the duality mappings the iterates are non-linear. Moreover, instead of the Banach space norm it is more common to use the Bregman distance for the convergence analysis. Due to these features of the problem, the analysis of the corresponding stochastic iterations requires a set of tools and ideas different from Hilbert space settings, and different from the deterministic setting.

In this work we will present a mathematical framework for stochastic gradient descent in Banach spaces, and the corresponding convergence analysis, for solving linear inverse problems. This is achieved by combining insights from Hilbert space theory with novel approaches from modern stochastic tools for non-linear stochastic optimisation. The theoretical findings shed further insights into the performance of the algorithm and open doors for a wide range of new methods that we aim to study in future work.

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LEARNED LANDWEBER ITERATION FOR THE TERAHERTZ TOMOGRAPHY

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Abstract

When talking about inverse problems deep learning and neural network based algorithms are an emerging field. We consider the inverse problem of terahertz tomography which is an imaging
technique for monitoring plastics and ceramics. Starting with the nonlinear eikonal equation as physical model we tackle the problem by a learned Landweber iteration. The eikonal equation can be seen as a high frequency approximation of the Helmholtz equation and, more generally, of the wave equation. Using the Landweber iteration we have to solve the partial differential equation for the forward operator in every step, but also need to compute the adjoint operator of the Fréchet derivative. We reduce the computing time by learning both, the forward operator as well as the adjoint of the Fréchet derivative.
Consequently, first we investigate the eikonal equation with respect to the theory of inverse problems. Then we integrate neural networks in the Landweber iteration. In the talk we try to combine the theory of inverse problems with the practical use of neural networks.

This is joint work with Marcel Mayr (SKZ Würzburg), Prof. Dr. Thomas Schuster and Dr. Anne Wald.

CONVERGENCE RESULTS FOR NONLINEAR TIKHONOV REGULARIZATION WITH OVERSMOOTHING PENALTY

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Abstract

For solving nonlinear problems in Hilbert scales, we present results on Tikhonov regularization with oversmoothing penalty term, i.e., the exact solution of the nonlinear problem does not belong to the domain of definition of the considered penalty functional. The focus of the present work is to provide convergence results for this setting, without assuming any smoothness of the solution of the considered nonlinear problem. This work continues previous studies, where convergence rates are provided for a priori- and a posteriori parameter choice strategies, respectively, under certain smoothness assumptions on the solution.

This is joint work with B. Hofmann (TU Chemnitz).

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GENERAL HEURISTIC RULE FOR CHOOSING REGULARIZATION PARAMETER IN TIKHONOV METHOD

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Abstract

We consider an operator equation

$$Au = f_*$$

where $A \in L(H,F)$ is the linear continuous operator between real Hilbert spaces H and F. We assume that instead of the exact right-hand side f_* we have only an approximation $f \in F$. To get regularized solution we consider Tikhonov method $u_\alpha = (\alpha I + A^* A)^{-1} A^* f$. Well-known heuristic rule for choosing regularization parameter $\alpha > 0$ is the quasioptimality principle, where the parameter is chosen as the global minimum point of the function $\psi_Q(\alpha) = \alpha \| du_\alpha / d\alpha \|$ on the set of parameters $\Omega = \{\alpha_j : \alpha_j = q\alpha_{j-1},\ j = 1,2,\ldots,M,\ 0 < q < 1\}$. Unfortunately, this rule is unstable in this sense that it often fails in case of heat-type problems. To get stable parameter choice rules we introduce modified quasioptimality criterion function in the form $\psi_{MQ}(\alpha_0) := \psi_Q(\alpha_0)$,

$$\psi_{MQ}(\alpha_j) := \max\left\{\psi_Q(\alpha_j), \left(d_{MD}(\alpha_j)/d_{MD}(\alpha_{j-1})\right)^{2/3} \psi_{MQ}(\alpha_{j-1})\right\},\ j = 1,2,\ldots,M,$$

where the function $d_{MD}(\alpha) = \| B_\alpha (Au_\alpha - f) \|,\ B_\alpha = \alpha^{1/2} (aI + AA^*)^{-1/2}$.

General heuristic rule. For the regularization parameter choose the parameter

$$\alpha_H = \max\{\alpha_{low}, \min\{\alpha_{MQ}, \alpha_{upp}\}\}.$$

For the parameter α_{MQ} we can take the global minimum point of the function $\psi_{MQ}(\alpha)$ on the set Ω or on the set $L_{\min}$, where $L_{\min}$ is the set of the local minimum points of the function $\psi_Q(\alpha)$. Another possibility is to find the parameter α_{MQ} using triangle area rule [1] with the function $\psi_{MQ}(\alpha)$.

For the parameter α_{low} we can take the global minimum point of the function $\psi_Q(\alpha)$ or the parameter, which is found by triangle area rule with the function $\psi_Q(\alpha)$. For the parameter α_{upp} we can take global minimum point of the functions $\psi_{HR}(\alpha) = \alpha^{-1/2} d_{MD}(\alpha)$ or $\psi_{DR}(\alpha) = \alpha^{-1/2} \| Au_\alpha - f \|$.

All rules defined in such way are stable and they differ from each other only in the accuracy of the rule and the complexity of algorithm.

This work was supported by Estonian Research Council team grant PRG864.

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CONVERGENCE ANALYSIS OF (STATISTICAL) INVERSE PROBLEMS UNDER CONDITIONAL STABILITY ESTIMATES

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Abstract

Conditional stability estimates require additional regularization for obtaining stable approximate solutions if the validity area of such estimates is not completely known. In this context, we consider ill-posed nonlinear inverse problems in Hilbert scales satisfying conditional stability estimates characterized by general concave index functions. For that case, we exploit Tikhonov regularization and provide convergence and convergence rates of regularized solutions for both deterministic and stochastic noise.

We further discuss apriori and a posteriori parameter choice rules and illustrate the validity of our assumptions in different model and real world situations.

A MIXED REGULARIZATION METHOD FOR ILL-POSED PROBLEMS

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Abstract

We propose a mixed regularization method for ill-posed problems. The method combines iterative regularization methods and parameter regularization methods effectively. First it applies iterative regularization methods in which there is no continuous regularization parameter to solve the normal equation of the ill-posed problem. Then continuous parameter regularization methods are applied to solve its residual problem. We consider the following system

$$T(x) = y^{\delta}, \qquad (1)$$

where $T : X \longmapsto Y$ is a bounded linear operator, X and Y are two Hilbert spaces, y^{δ} is the known data with a noise level δ. After (1) has been solved by an iterative regularization method yielding the solution x^{it}, there is still a residual term $y^{\delta} - Tx^{it}$. Then we apply a parameter regularization method to solve the residual equation. The final solution is the summation of the solutions of the iterative regularization method and the parameter regularization method and can be expressed as

$$x^{mix} = x^{it} + g_{\alpha}(T^{*}T)T^{*}(y - y^{\delta}), \qquad (2)$$

where x^{mix} is the solution of the mixed regularization method, g_{α} is at least piecewise continuous function associated with α and its detail description can be found in [1]. When the noise level δ is known, we can choose the regularization parameter by a posterior method such as ME-rule [2]. The ME-rule chooses the smallest regularization parameter $\alpha = \alpha_{ME}$ for which we can guarantee the ME-property: the error of the solution in this stage is monotonically increasing for $\alpha \in [\alpha_{ME}, \infty)$. We have proved that the solution x^{mix} obtained by the mixed regularization method is better than x^{it} by the iterative regularization method.

Our method is a combination of iterative regularization methods and parameter regularization methods. The iterative regularization methods are such methods that they do not need continuous regularization parameters whereas parameter regularization methods need. In iterative methods such as the Landweber method, the stopping rule is crucial since the phenomenon of semi-convergence. The mixed regularization method can be considered as an improvement of the continuous parameter regularization method. In continuous parameter regularization methods, the choice of the regularization parameter is crucial because the solution depends on the parameter sensitively. Our theoretical analysis and many numerical examples show that the new mixed regularization method can reduce the sensitivity of the regularization parameter and improve the solution of iterative regularization methods. It can reach the optimal convergence order under a much wider range.

This is a joint work with Hui Zheng.

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SPLITTING FOR JUMP-DIFFUSION CALIBRATION IN FINANCIAL OPTION MODELS

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Abstract

Dupire's local volatility model is extensively used and well-recognized for hedging and option pricing in financial markets [1]. The inverse problem consists in recovering the time and space varying diffusion coefficient in a parabolic partial differential equation from limited data. It is known that this corresponds to an ill-posed problem. Such ill-posed character of local volatility surface calibration from market prices requires the use of regularization techniques either implicitly or explicitly. Such regularization techniques have been widely studied for a while and are still a topic of intense research [2-5]. We have employed convex regularization tools and recent inverse problem advances to deal with the local volatility calibration problem. See [2-6] and references therein.

This talk concerns the calibration of Dupire's model in the presence of jumps. This leads to an integro-differential equation whose parameters have to be calibrated so as to fit market data. We present a splitting strategy to identify simultaneously the local-volatility surface and the jump-size distribution from quoted European prices. The underlying stochastic model consists of a jump-diffusion driven asset with time and price dependent volatility.

Our approach uses a forward Dupire-type partial-integro-differential equation for the option prices to produce a parameter-to-solution map. The ill-posed inverse problem for such a map is then solved by means of a Tikhonov-type convex regularization. We present numerical examples that substantiate the robustness of the method both for synthetic and real data.

This talk is partially based on joint work with Vinicius Albani (UFSC, Brazil) published in [6]. The research was supported by FAPERJ under grant E26/202.927 and by Khalifa University under FSU-2020-09.

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MINISYMPOSIUM
M3: Tomographic Inverse Problems

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Tomography has revolutionized diagnostic medicine, nondestructive evaluation and materials science. By nature, tomography is an inverse problem—recovering information about the structure or properties of an object using indirect data. Modern modalities include novel methods in X-ray CT, hybrid imaging, multi-modal imaging, multi-energy CT, ultrasound, Compton CT, and time-dependent problems. Each modality generates novel mathematics and new algorithms, many of which will be presented in this minisymposium.

We will bring together young and established researchers from the inverse problems community to present their recent work and to foster discussion among participants.

INVERTIBLE DISTRIBUTIONS, MEAN VALUE OPERATORS AND SYMMETRIC SPACES

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Abstract

Integral transforms that are translation-invariant can be thought of as convolution operators. Therefore, to study the mapping properties of such integral transforms, it is often useful to study convolution operators in general. One such interesting type of translation-invariant integral transform is the fixed-radius mean value operator M^r on $\mathbf{R}^n$. More generally, one can study orbital integrals on homogeneous manifolds as either convolution operators or group-invariant Radon transforms.

In the late 1950's and early 1960's Leon Ehrenpreis introduced the idea of slowly decreasing functions to describe the Fourier transforms of compactly supported distributions S on $\mathbf{R}^n$ for which the convolution operator $f \to f * S$ has a fundamental solution and is surjective on various function and distribution spaces. We will discuss this criterion in detail and some of its applications to mapping properties of mean value operators, as well as some recent related results on wave equations as well as extensions to compact and noncompact symmetric spaces.

ESTIMATING HYPERPARAMETERS IN HIERARCHICAL BAYESIAN LINEAR INVERSE PROBLEMS

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Abstract

We consider a hierarchical formulation of the Bayesian inverse problems in which the forward operator is assumed to be linear, and we treat unknown parameters associated with the noise and the prior as hyperparameters. We marginalize over the unknown parameters and compute the maximum a posteriori estimator over the marginalized distribution. We use Krylov subspace methods to accelerate the computations of the nonlinear optimization technique.

We demonstrate the performance of our approach on several synthetic test problems.

ALGORITHMS FOR THE COMPTON SCATTER IMAGING IN POSITRON EMISSION TOMOGRAPHY

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Abstract

We investigate a forward model of the Compton scatter in PET using the SSS technique [1] that estimates a single scatter in the detector pair (A,B) as an integral over the total scatter volume V :

$$S_V^{AB} = \int_V dV \frac{\sigma_{AS}\sigma_{SB}}{4\pi |AS|^2 |SB|^2} \frac{\mu}{\sigma_C} \frac{\partial \sigma_C}{\partial \Omega} (I^A + I^B); I^A = e^{-(\int_A^S \mu dl + \int_S^B \mu' dl)} \int_A^S f dl; I^B = e^{-(\int_A^S \mu' dl + \int_S^B \mu dl)} \int_S^B f dl. \quad (1)$$

Here, σ_{AS} and σ_{BS} are the geometric cross-sections of the detectors A and B, f is the emitter activity, $\mu = \mu(E,S)$ is the linear attenuation coefficient depending on the energy E and the scatter point S, and $\partial\sigma_C / \partial\Omega$ is the differential cross-section. Primed and unprimed quantities are evaluated at the scattered and unscattered photon energies, respectively. Equation (1) is symmetrical in terms of A and B so that the primary photons are recorded both by A and B. We investigate a one-sided version of (1), where only the detector A is tuned to counting the primary photons, that is, we set $I^B = 0$ within idealized assumptions of the excellent energy resolution and detection efficiency for the detectors A and B. Then (1) can be reduced [2] (in spherical coordinates with origin in A) to:

$$\xi_\theta^{AB} = \int_0^\theta d\varphi \frac{\cos\varphi \cos(\varphi - \theta)}{4\pi |AB|} \int_0^{2\pi} d\psi \frac{\mu(\psi, \varphi, |AS|)}{\sigma_C} \frac{\partial \sigma_C}{\partial \Omega} e^{-(\int_A^S \mu dl + \int_S^B \mu' dl)} \int_0^{|AS|} f(\psi, \varphi, r) dr. \quad (2)$$

The kernel support has a spindle shape, and the kernel larger values are concentrated around the detector A, rapidly decreasing in the direction to the detector B. It also rapidly decreases in the lateral direction, from the vicinity of the central line AB to the kernel borders, so that the kernel gives larger weights to the activity for the areas close to A and smaller weights to the kernel domain around B. There emerges additional information from a scatter about the activity hot spots located close to the patient's body periphery. We reduce (2) to the slice-by-slice convolution distance-dependent blurring model of the projection formation provided that the attenuation map is assumed to be constant. This model has been proven earlier as invertible [3]. We show that the three-dimensional slice-by-slice filtered backprojection algorithm developed in [3] is applicable to the scatter data inversion. Because of the idealized assumptions involved in the derivation of the analytical scatter projector (2), we validate this model with Monte Carlo simulation. The numerical comparative study has been performed using a digital cylindrical phantom filled with water containing spherical activity sources. We present the results illustrating the numerical experiments conducted.

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SPARSITY-BASED TECHNIQUES FOR HYBRID IMAGING MODALITIES WITH MISSING LOW FREQUENCIES

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Abstract

Several novel hybrid imaging modalities rely on propagating sound waves to deliver spatial information about the properties of the object. Traditional image reconstruction techniques assume that an infinitely wide range of acoustic wavelengths is available. However, commonly utilized piezoelectric transducers emit/receive only a narrow band of frequencies, with both very long and very short wavelengths missing. This leads to strong distortions in the reconstructed images.

We will investigate the use of sparsity-inducing techniques (such as, for example, L1-regularization) to overcome this problem.

This is a joint work with N. Do and P. Hoskins.

NOVEL INVERSE PROBLEMS IN COMPTON TOMOGRAPHY

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Abstract

In this talk, we will describe novel luggage testing methods using Compton CT. We will analyze their microlocal properties and describe possible added artifacts and visible and invisible singularities of the objects.

We will outline reconstruction methods that suppress the artifacts and provide reconstructions illustrating our results.

NUMERICAL RECONSTRUCTION FROM THE FOURIER TRANSFORM ON THE BALL VIA PROLATE SPHEROIDAL WAVE FUNCTIONS

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Abstract

We implement numerically formulas of [1] for finding a compactly supported function v on R^d, $d \geq 1$, from its Fourier transform Fv given within the ball B_r.

For the one-dimensional case, these formulas are based on the theory of prolate spheroidal wave functions, which arise, in particular, in the singular value decomposition of the aforementioned band-limited Fourier transform for $d = 1$.

In multidimensional case, these formulas also include inversion of the classical Radon transform.

This talk is based on the joint work with M. Isaev and R.G. Novikov.

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RECONSTRUCTION OF SPARSE LOG-CONDUCTIVITY IN CURRENT DENSITY IMPEDANCE IMAGING

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Abstract

In this talk, we present a new approach to reconstruct sparse log-conductivities in an object through current density impedance tomography (CDII). CDII is a non-invasive hybrid imaging modality where voltage is passed to electrodes placed across an object and the corresponding interior electric field is measured. Using these measurements, one tries to reconstruct the interior of the object. Unfortunately, existing mathematical reconstruction algorithms suffer from loss of contrast in the images. We, thus, use an alternate method comprising of sparse optimization and edge-enhancing anisotropic diffusion to obtain images with high quality.

Several numerical experiments demonstrate the superiority

SYMMETRY FROM SECTIONAL INTEGRALS FOR CONVEX DOMAINS

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Abstract

Let Ω be a bounded convex domain in R^n ($n \geq 2$). In this work, we prove that if there exists an integrable function f such that it's Radon transform over $(n-1)$-dimensional hyperplanes intersecting the domain Ω is a function G depending on the distance to the nearest parallel supporting hyperplane to Ω, then Ω is a ball and f is radial depending on certain assumptions on G. As a consequence we show that constants are not in range of Radon transform of integrable functions in dimensions $n \geq 3$.

This work jointly done with a fellow PhD student Mr. Ramya Dutta.

FUNCTIONAL-ANALYTICAL METHODS IN ACOUSTIC INVERSE ROBLEMS

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Abstract

In this report possibilities of functional algorithms [1–3] for purposes of 2D and 3D acoustic tomography are discussed. Initially, these algorithms have been developed in quantum mechanics. To understand applicability of these methods for acoustics inverse problems the detail investigations based on numerical modeling were implemented [4-6]. For acoustic applications it is important that these algorithms take into account the multiple scattering processes and do not require iterations or additional regularizations. Moreover, the joint reconstruction of both scalar (i.e. sound speed, absorption coefficient, density) and vector (i.e. flows) characteristics of the medium is possible in a single tomography scheme, without additional separation of influence of inhomogeneity components on the scattering data.

In 2D case the tomography scheme for joint reconstruction of spatial distributions of sound speed, density, absorption together with recovering the frequency dependence of the absorption is numerically studied. It is shown, that the interference resistance of this algorithm is quite high that indicates the possibility of its practical application for medical purposes (for example, in the early diagnostics of cancer in mammology), taking into account the abilities of real tomographic devices.

Numerical results of 3D reconstruction are also presented for the purposes of ocean acoustic tomography. In this case 3D inverse problem is approximated by a set of 2D ones which take into account multi-channel scattering effects. These results are perspective for the ocean tomography when nonadiabatic propagation of acoustic modes in ocean waveguides should be taken into account. Results of numerical reconstructions of 3D ocean inhomogeneities based on the 2D multi-channel functional-analyticla algorithm are presented, which show high resolution and acceptable for practical applications the interference resistance of this algorithm.

The comparative numerical investigation of iterative algorithm [7] and the considered functional-analytical algorithm for solving 2D acoustic tomography problem is also considered. For the case of the two-dimensional scalar Helmholtz equation, the efficiency of the iterative algorithm for the reconstruction of medium-strength scatterers and the advantages of the functional-analytical approach for the reconstruction of strong scatterers with parameters typical for acoustic tomography problems are demonstrated [8]. The main advantage of the iterative approach in comparison with the functional-analytical algorithm is the ability to make reconstruction from incomplete scattering data. This is especially important in 3D inverse problems.

The obtained results of numerical investigations of noise immunity and resolution show that the functional-analytical algorithm can be applied in practical acoustic tomography problems, as well as the possibility of developing new tomographic schemes based on iterative and functional-analytical approaches.

The reported study was funded by RFBR and CNRS, project number 20-51-15004.

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MINISYMPOSIUM
M4: Modern Challenges in Inverse Problems Including Boundary Rigidity, Microlocal Analysis and Cloaking

Minisymposium dedicated to the 70th anniversary of an outstanding expert in inverse problems, Walker Family Endowed Professor of Mathematics University of Washington, Gunther Uhlmann

Organizers:
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Professor Gunther Uhlmann has made fundamental contributions for decades to microlocal analysis and applications, the solution of important inverse problems including Electrical Impedance Tomography (EIT) also called Calderon's problem, travel time tomography, also called boundary rigidity and lens rigidity, integral geometry, and inverse problems arising in general relativity. He and collaborators have done pioneering work in proposing transformation optics, a method to make objects invisible to electromagnetic waves, acoustic waves, matter waves, and other types of waves.

This minisymposium celebrates the anniversary of Professor Uhlmann, his many fundamental and insightful contributions to inverse problems and partial differential equations, and his pioneering work in the mathematics of boundary rigidity, microlocal analysis and cloaking. The methods pioneered by Prof. Gunther Uhlmann are essential contributions to almost all areas of inverse problems.

THE SOLID-FLUID TRANSMISSION PROBLEM

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Abstract

I will discuss recent joint work with Plamen Stefanov ([1]) on the microlocal analysis of the transmission problem at an interface between an isotropic linear elastic solid and an inviscid fluid. This problem is motivated from geophysics: there one is interested in understanding the propagation of seismic waves in the interior of the Earth, which consists of several solid and fluid layers. When a seismic wave meets the interface between two layers, part of its energy is reflected (possibly with mode conversion), and, if the angle of incidence is not too large, part of it is transmitted to the other side of the interface. In a spirit similar to previous works studying microlocally the transmission problem at the interface between two solids ([2,3,4,5]), our goal is to understand reflection, transmission and mode conversion of singularities at the interface between a solid and a fluid. For simplicity, we consider the case of two layers, with the fluid layer being enclosed by the solid one. The two media are described by a system of PDEs modeling the displacement in the solid and pressure-velocity in the fluid, with these quantities being coupled at the interface by transmission conditions. We show well posedness for the system, and construct and justify a parametrix for it (approximate solution up to smooth error) using geometric optics.

As an application of our study, we consider the inverse problem of recovering the wave speeds in the two layers and the material density in the solid outer layer from the Neumann-to-Dirichlet map for the solid-fluid system corresponding to the exterior boundary.

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SOBOLEV ESTIMATES FOR MULTILINEAR RADON TRANSFORMS VIA PARTITION OPTIMIZATION

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Abstract

I will describe a method for obtaining Sobolev space bounds for multilinear generalized Radon transforms and Fourier integral operators by optimizing over collections of linear estimates. This approach was motivated by problems in geometric measure theory/combinatorics, and I will give examples of these applications, but this technique might be applicable for other problems involving multilinear operators, such as Born expansions.

This is joint work with Alex Iosevich and Krystal Taylor.

SIMULTANEOUS RECOVERY OF ATTENUATION AND SOURCE DENSITY IN SPECT

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Abstract

I will discuss some recent results about simultaneous recovery of the attenuation and source density in the SPECT inverse problem. Assuming the attenuation is piecewise constant and the source density piecewise smooth we show that, provided certain conditions are satisfied, it is possible to uniquely determine both. I will also discuss a numerical algorithm that allows for determination of both parameters in the case when the range of the piecewise constant attenuation is known.

This is based on joint work with Philip Richardson.

INVERSE PROBLEMS FOR FINITE GRAPHS AND CLOAKING

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Abstract

We study the inverse problem of determining a finite weighted graph having vertexes X and edges E from the observations made on a finite subset of vertexes subset $B \subset X$. The observations we consider are
1) the source-to-solution map on a vertex subset $B \subset X$ for heat equation on a graph,
2) eigenvalues and restrictions of eigenfunctions on $B \subset X$,
3) observations of a random walk moving on a graph.

We solve these problems when the graph satisfies the Two-Points Condition. This condition is valid for trees and perturbations of finite lattices. We also discuss how the studied problems are related to Finite Element Method, resistor networks, and inverse problems on manifolds and invisibility cloaking.

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INVERSE TRANSPORT AND DIFFUSION PROBLEMS IN PHOTOACOUSTIC IMAGING WITH NONLINEAR ABSORPTION

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Abstract

Two-photon photoacoustic tomography (TP-PAT) is a noninvasive optical molecular imaging modality. It aims at inferring the two-photon absorption property of heterogeneous materials, such as biological tissues, from photoacoustic measurements. In this talk, we will discuss the work [1] motivated by TP-PAT on inverse coefficient problems for a semilinear radiative transport equation and its diffusion approximation with internal data. These data are functionals of the coefficients and the solutions to the equations. Uniqueness and stability results of the inverse problems as well as its uncertainty quantification will also be discussed.

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WAVE PROPOGATION INSIDE TRANSPARENT SCATTERERS AND APPLICATIONS

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Abstract

Consider the time-harmonic wave scattering from an inhomogeneous medium scatterer. If transparency/invisibility occurs, the incident and total wave fields form a pair of transmission eigenfunctions. We shall discuss our recent discovery on the spectral geometry of transmission eigenfunctions as well as its applications to super-resolution imaging, artificial mirage and pseudo polariton resonances.

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INTERSECTION RIGIDITY FOR SIMPLE RIEMANNIAN MANIFOLDS

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Abstract

Consider a compact Riemannian manifold with boundary. For each pair of inward pointing unit vectors at the boundary, there is a corresponding pair of geodesics through the manifold. The "intersection data" is the collection of all pairs of inward pointing directions whose geodesics have non-empty intersection. This data is equivalent to forgetting the travel time information from the broken scattering relation [1]. We show that the intersection data for a simple Riemannian manifold determines its isometry class when the dimension of the manifold is at least 2.

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MAPPING PROPERTIES OF X-RAY TRANSFORMS NEAR CONVEX BOUNDARIES

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Abstract

On a Riemannian manifold with boundary, the X-ray transform integrates a function or a tensor field along all geodesics through the manifold. The reconstruction of the integrand of interest from its X-ray transform is the basis of important inverse problems with applications to seismology and medical imaging.

The inversion of the X-ray transform is often done by inverting the normal operator (composition of the X-ray transform and its adjoint, the "backprojection" operator). This requires the design of appropriate Sobolev scales or Frechet spaces over the manifold and the manifold of its geodesics, compatible with an appropriate formulation of forward and backward mapping properties for the X-ray transform, the backprojection operator, and their composites. For example, a landmark result by Pestov and Uhlmann, paving the way toward their 2005 result on the boundary rigidity of simple surfaces [1], was the design of a good Frechet setting where the backprojection operator is surjective.

In this talk, I will review recent results [2,3,4,5,6,7] attempting to shed additional light on the (forward and backward) mapping properties of the X-ray transform and its normal operator(s) on convex, non-trapping manifolds.

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RECENT DEVELOPMENTS IN PHOTOACOUSTIC TOMOGRAPHY

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Abstract

Photoacoustic Tomography is a promising hybrid medical imaging modality that can generate high-resolution and high-contrast images by exploiting the coupling of electromagnetic pulses (in the visible region) and ultrasound waves via the photoacoustic effect. One step of the mathematical problem consists in the determination of an acoustic source from boundary observations. There exist several ways of addressing the reconstruction of the initial conditions, with varying performances depending highly on the underlying assumptions on the acoustic properties of the body, in particular, on the hypotheses imposed on the speed of sound. Some examples include integral formulae and eigenvalue expansion methods mainly for homogeneous backgrounds, while time-reversal based approximations and iterative reconstruction procedures are better suited for acoustically heterogeneous media.

In the last 13 years, many new results have stemmed from the crucial work of Stefanov and Uhlmann between 2009 and 2013 [1,2,3], which provided a deep understanding of the underlying geometrical problem. Their microlocal approach combined with functional analysis and geometry allowed them to answer the fundamental questions of uniqueness and stability, and to obtain an iterative reconstruction formula in the form of a Neumann series. Since then, researchers have been able to extend and generalize to different settings their techniques and ideas, however, many open questions still remain to be answered, especially in the more intricate problem of simultaneous determination of the initial source and the sound speed with a single boundary measurement.

In this talk, I will review the main ideas introduced in [1,2,3] and I will talk about some of the latest developments on this problem, in particular the results obtained in [4] related to the partial data case. I will also address the aforementioned joint determination problem of recovering the initial source and the sound speed with a single boundary measurement. We will review previous results as well as recent progresses made in collaboration with Sebastian Acosta [5].

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FORMALLY DETERMINED INVERSE PROBLEMS FOR HYPERBOLIC PDES

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Abstract

We describe two stability results for formally determined inverse problems for hyperbolic PDEs. Our first result is Lipschitz stability for the fixed angle scattering problem for the operator $\Box + q(x)$, where the data consists of the medium response to incoming plane waves from two opposite directions.

This is joint work with Mikko Salo. Our second result is Lipschitz stability for the fixed angle scattering problem for the operator $\Box + q(x)$, where the data consists of the medium response to plane waves coming from the same fixed direction but with different time delays.

This is joint work with Venky Krishnan and Soumen Senapati. For both these problems there are also results for the operator $(\partial_t -a)^2 - (\nabla -b)^2 + c$, by Salo et al for the time independent case, and by Krishnan, Rakesh and Senapati for the time independent case.

We also have similar results with point sources instead of plane wave sources.

INSTABILITY MECHANISMS FOR INVERSE PROBLEMS

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Abstract

Many inverse and imaging problems, such as image deblurring or electrical/optical tomography, are known to be highly sensitive to noise. Such problems are called ill-posed or unstable, as opposed to being well-posed (a notion introduced by J. Hadamard in 1902). Instability lies at the heart of inverse problems research and much of the literature addresses various aspects of it.

The inherent reason for instability is easy to understand in linear inverse problems like image deblurring. For more complicated nonlinear imaging problems the instability issue is more delicate. We will discuss a general framework for understanding instability in inverse problems based on smoothing/compression properties of the forward map together with estimates for entropy and capacity numbers in relevant function spaces. The methods apply to various inverse problems involving general geometries and low regularity coefficients.

This talk is based on joint work with Herbert Koch (Bonn) and Angkana Rüland (Heidelberg).

ON THE RANGE OF THE PLANAR X-RAY TRANSFORM OF SYMMETRIC TENSORS ON THE FOURIER LATTICE OF THE TORUS

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Abstract

I will present new necessary and sufficient conditions on the Fourier coefficients of a function on a torus to be in the range of the X-ray transforms of symmetric two tensors compactly supported in the unit disc.

The conditions can be construed to define a deficiency index for the X-ray data.

References

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ESTIMATE THE SIZE OF AN INCLUSION IN A BODY WITH COMPLEX CONDUCTIVITY USING FINITE NUMBER OF MEASUREMENTS

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Abstract

In the inverse boundary value problems of the second order elliptic equation with complex conductivity, we derive estimates for the volume fraction of an inclusion whose physical parameters satisfy suitable gap conditions. Lower and upper bounds are obtained by using three pairs of boundary measurements. We accomplish this with the help of the "translation method" which consists of perturbing the minimum principle associated with the equation by either a null-Lagrangian or a quasi-convex quadratic form.

This is a joint work with Catalin Carste.

INTEGRAL GEOMETRY PROBLMES IN LORENTZIAN GEOMETRY

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Abstract

We consider the light ray transform on Lorentzian manifolds, which concerns the integral of functions along light-like (or null) geodesics. The transform is related to the Radon transform or geodesic ray transform in the Riemannian setting. An outstanding question is what information regarding the function can be recovered from the light ray transform. In this talk, we discuss recent developments, including injectivity, stability results and microlocal properties of the transform. Also, we discuss the applications of the transform to inverse problems in cosmology and Lorentzian geometry. The talk is partly based on references [1-3].

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INVERSE PROBLEMS FOR NONLINEAR PDES

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Abstract

In this talk, I will demonstrate the higher order linearization approach to solve several inverse boundary value problems for nonlinear PDEs modeling nonlinear electromagnetic optics including nonlinear time-harmonic Maxwell's equations, nonlinear magnetic Schrodinger equation and its fractional version. The problem will be reduced to solving for the coefficient functions from their integrals against multiple linear solutions. We will focus our discussion on different choices of linear solutions.

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MINISYMPOSIUM
M5: Theoretical and Numerical Results in Geometric Inverse Problems for PDEs

Organizers:
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In recent years, the interest for the analysis and simulations of inverse problems for Partial Differential Equations (PDE's) of many kinds has grown a lot. This is motivated for their relevance in many applications: elastography and medical imaging, seismology, potential theory, ion transport problems or chromatography and finances, for example. The study of such type of problems includes theoretical results on the uniqueness, stability and numerical approximations.

The purpose of this minisymposium is to show and share different analytical and computational methods for inverse problems that could be based in the problems that arise from Science, Medicine or Technology.

GEOMETRIC INVERSE PROBLEMS FOR THE BURGERS EQUATION AND RELATED SYSTEMS

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Abstract

The analysis and solution of geometric inverse problems has recently increased a lot because of their importance in many applications: elastography and medical imaging, seismology, potential theory, ion transport problems or chromatography and other similar fields.

In this talk we will deal with one-dimensional inverse problems concerning the Burgers equation and some related nonlinear systems (involving heat effects and/or variable density). In these problems, the goal is to find the size of the spatial interval from some appropriate boundary observations of the solution. Depending on the properties of the initial and boundary data, we will see uniqueness and non-uniqueness results. We will also show numerical solutions to the inverse problems mentioned above, computing accurate approximations of the interval sizes.

This is a joint research project in collaboration with Anna Doubova, Enrique Fernández-Cara and Masahiro Yamamoto.

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INVERSE PROBLEMS FOR PARABOLIC EQUATION IN NETWORKS WITH LOOPS

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Abstract

In this talk we prove the observability of parabolic equations on networks with loops, and use this to obtain results related with inverse problems. We show that the observability of the entire network can be achieved under certain hypotheses about the position of the observation domain. This is done using a novel Carleman inequality. The main difficulty we tackle, due to the existence of loops, is to avoid entering into a circular fallacy, notably in the construction of the auxiliary function for the Carleman inequality. This is overcome as the hypotheses allow setting from which end the heat on the edge is observed on the edges with empty intersection with the observation domain.

ADAPTIVE FINITE ELEMENT METHOD FOR SOLUTION OF INVERSE AND ILL-POSED PROBLEMS

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Abstract

An adaptive finite element method for application to solution of coefficient inverse and some ill-posed problems will be discussed. These problems usually are formulated as optimal control problems for minimization of the Tikhonov's regularization functional [1].

The main idea of an adaptive finite element method is following: it allows to obtain good accuracy of the computed solution of inverse or ill-posed problem via local adaptive mesh refinements. To be able use this idea, the Tikhonov functional is minimized on a sequence of locally refined finite element meshes. These meshes are refined based on the posteriori error estimates in the obtained finite element reconstruction or in the error in the Tikhonov's functional. We will outline main steps in derivation of a posteriori error estimates. Based on these estimates, adaptive finite element algorithms will be formulated and their work will be demonstrated on the real-life problem of monitoring of hyperthermia [2].

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MULTILEVEL CONTROL

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Abstract

We discuss the multilevel control problem for dynamical systems, consisting in designing a piecewise constant control function taking values in a finite-dimensional set. We start by providing a complete characterization of multilevel controls through an optimal control approach, based on the minimization of a suitable cost functional. In this manner we build optimal multilevel controls and characterize the time needed for a given ensemble of levels to assure the controllability of the system [1]. In a second moment, as a practical application of our results, we employ the multilevel control strategy to efficiently solve the Selective Harmonics Modulation problem in power electronics engineering [2].

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NUMERICAL RECONSTRUCTION OF THE CONDUCTIVITY IN THE 3-D CALDERON PROBLEM USING THE BORN APPROXIMATION

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Abstract

Reconstruction in the three-dimensional Calderon inverse conductivity problem can be reduced to the study of the inverse boundary problem for Schrödinger operators $-\Delta + q$. A suitable linearization of the inverse problem allows to define the so-called Born approximation. As described in [1], this provides a good reconstruction only for the high frequencies of q. In a recent work [2], we established a new formula relating this Born approximation with the Dirichlet to Neumann map and the momenta of q. Based on this formula, we give a new convergent iterative algorithm that improves the numerical reconstruction given by the Born approximation. We illustrate the process with numerical experiments for radial and non-radial potentials.

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TRANSCRANIAL DIRECT CURRENT STIMULATION: ESTIMATION OF INDIVIDUALLY TARGETED USING THE HEAD GEOMETRY

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Abstract

Electrical stimulation to modify brain function has been studied since Bartholow's pioneering work [1] in 1874. Nowadays, among the many modern non-invasive neurostimulation techniques, transcranial direct current stimulation (tDCS) stands out, which has been applied to reduce the symptoms of post-stroke motor and language deficits, associated with depression, chronic pain, memory loss and tinnitus. The required equipment is simple, versatile, easy applied and relatively low in cost. Furthermore, its adverse effects are rare, transient, and well-tolerated, in general, ranging from itching, tingling, and burning sensations to redness on the stimulated areas, headaches, nausea, and insomnia [2].

However, the specialized literature indicates that results strongly depend on the adopted protocol [3] (such as electrode positions, duration of stimulation, number of sessions, intensity of current, etc.) and the head and brain's anatomy [4], indicating a need to shift from a one-size-fits-all pacing approach to an individualized induced e-field strength.

The associated inverse problem, therefore, is given a prescribed electric field distribution, defining the electrode placement and the required current intensity, accounting for head and brain configurations.

In this work we used the open-source software SimNIBS (Simulation of Non-invasive Brain Stimulation) in order to consider a realistic finite element head model. Then, we estimated the electrode placement taking into account the topological derivative [5] for a suitable shape functional, considering several cranial geometries. Finally, we performed the hierarchical clustering dendrogram, associated to the single-linkage clustering [6], in order to estimate individualized tDCS parameters.

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A RESISTANCE METHOD FOR A FLUID-OBSTACLE INVERSE PROBLEM

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Abstract

We present an obstacle inverse problem in fluids. We analyze it from the point of view of recovering the boundary obstacle from velocity measurements and unknown pressure. First we show how to approximate the geometrical inverse problem with a parameter identification problem and then we establish some uniqueness and stability results using Carleman inequalities.

The problem is motivated from cardiac valve detection using MRI-4D flow measurements.

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MINISYMPOSIUM
M6: Generalized Radon Transforms and Applications

Organizers:
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The aim of this mini-symposium is to gather scientists working in the field of imaging inverse problems soluble by generalized Radon transforms. There has been many advances by various groups over the last decade. It is of interest to provide an overview of progress in this research field as well as related fields. Applications range from imaging in seismic, medical diagnostics, industrial non-destructive evaluation and control to detection-identification in monitoring nuclear safety, homeland security and ecological survey.

It is expected that participants would have the opportunity to meet each other, exchange ideas and possibly start collaborative work.

THE MODULO RADON TRANSFORMS AND ITS INVERSION

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Abstract

In this paper, we introduce the Modulo Radon Transform (MRT) which is complimented by an inversion algorithm. The MRT generalizes the conventional Radon Transform and is obtained via computing modulo of the line integral of a two-dimensional function at a given angle. Since the modulo operation has an aliasing effect on the range of a function, the recorded MRT sinograms are always bounded, thus avoiding information loss arising from saturation or clipping effects. This paves a new pathway for imaging applications such as high dynamic range computed tomography. By capitalizing on the recent results on Unlimited Sensing architecture, we prove that the Modulo Radon Transform can be inverted when the resultant (discrete/continuous) measurements map to a bandlimited function.

Thus, the MRT leads to new possibilities for both conceptualization of inversion algorithms as well as development of new hardware

STRUCTURED ULTRASOUND-MODULATED OPTICAL TOMOGRAPHY

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Abstract

Ultrasound-modulated optical tomography (UOT) is an imaging technique which couples light and ultrasound (US) in order to perform in-depth optical imaging of highly scattering media [4, 8, 7, 9]. A potential application is millimeter to centimeter-deep imaging of living tissues for medical diagnosis. Traditional images are obtained using MHz-range few-cycle Focused Waves (FW) as illustrated in Fig 1(a). By instead using Plane-Waves (PW) to tag photons, we can reconstruct an image using the filtered backprojection method [2, 1, 5]. Because of angular limitation however, the measurement suffers from a drastic loss in lateral spatial resolution. Here, we present a new structured ultrasonic plane wave UOT method that allows partial recovery of the resolution. We will introduce a **generalization of the Fourier Slice Theorem along with a generalized filtered backprojection formalism**.

The method is successfully tested on simulated and experimental data [3].

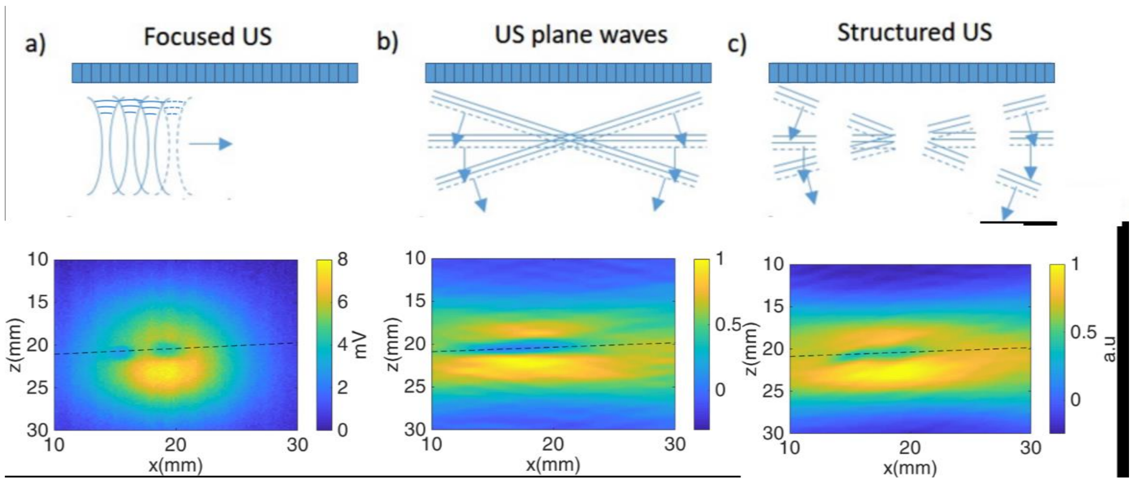

Fig. 1 : Experimental images of two absorbing inclusions obtained using respectively FW(a), PW(b), and SW ultrasonic waves(c). All images can be found with extensive descriptions in [3].

PW imaging consists in tagging photons using multiple a few-cycle US pulses emitted at different angles as represented in 1(b). As illustrated by the image below, the reconstruction quality in the lateral direction is poor because the angular span is limited by the acoustic probe directivity. PW-UOT remains nevertheless a powerfull alternative to FW when fast acquisition speed is required [6].

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ON RANGE CONDITION OF THE X-RAY TRANSFORM IN $\mathcal{R}^n$

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Abstract

Consider the problem of the range description of the tensor x -ray transform. Specific issue is to minimize the number of differential equations that describe the range.

In this study a geometrical interpretation of the range condition and related John differential operator is given. As a corollary, it is proved that the range of the m-tensor x-ray transform in $\boldsymbol{R}^n$ can be described by $\binom{n+m-2}{m+1}$ differential equations.

RAY TRANSFORM PROBLEMS ARISING FROM SEISMOLOGY ON MARS

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Abstract

We discuss recent uniqueness results in the mathematical theory of inverse problems and their relation to seismic investigations of Mars using NASA's InSight lander. Constrained by severely limited data, one must be careful in drawing conclusions from measurements.

We discuss some of the mathematical inverse problems, especially in ray tomography, that arise from this setting.

SPECTRAL COMPTON CAMERA IMAGING OF POLYCHROMATIC GAMMA-RAY SOURCES

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Abstract

In this work, we investigate spectral image reconstruction for Compton camera gamma-ray imaging. The forward model is defined to take into account incomplete absorption and to accurately describe the acquisition process at each energy. Total variation regularization can be added to compensate for low dose.

We present a novel pre-conditioned algorithm that allows an important reduction of the number of TV iterations. This algorithm can be applied to other tomography problems where the sensitivity matrix has important variations through the reconstructed volume, for instance in CBCT.

SOME GENERALIZED RADON TRANSFORMS INSPIRED FROM IMAGING TECHNOLOGY

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Abstract

We discuss some Generalized Radon Tranforms (GRT) which arise from various detection technologies of ionizing radiation, electromagnetic radiation or sound waves. They encompass the great majority of the relevant Radon transforms in imaging applications, namely the reconstruction of hidden structures of interest. Primary (or undeviated) radiation detection involves the well known classical Radon transform in Computed Tomography. However detection of transmitted radiation with Compton scattering over illuminated objects, at chosen energies leads to generalized Radon transforms on various families of *Circular Arcs* in the so-called Compton Scatter Tomography.

In addition, Emission tomography modalities such as SPECT or PET requires anew the classical Radon transform but Compton Scatter Emission Imaging introduces Conical Radon transform in three dimensions and the *V-line* Radon transform in two dimensions. Photo-Acoustic-Tomography (PAT) (resp. SAR) imaging uses Radon transforms on *Spheres* (or *Circles*) centered on a fixed circle (resp. a fixed line).

The main properties of the inversion formulas of these GRTs and their illustrating numerical simulations are presented to support their abilities for useful imaging applications (medical diagnostics, industrial non-destructive evaluation, security, cultural heritage object imaging, etc.).

This is a joint work with Cécilia Tarpau, Javier Cebeiro, T.T. Truong.

AUGMENTED NETT REGULARIZATION OF INVERSE PROBLEMS

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Abstract

We propose aNETT (augmented NETwork Tikhonov) regularization as a novel data-driven reconstruction framework for solving inverse problems. An encoder-decoder type network defines a regularizer consisting of a penalty term that enforces regularity in the encoder domain, augmented by a penalty that penalizes the distance to the data manifold. We present a rigorous convergence analysis including stability estimates and convergence rates. For that purpose, we prove the coercivity of the regularizer used without requiring explicit coercivity assumptions for the networks involved. We propose a possible realization together with a network architecture and a modular training strategy. Applications to the inversion of the Radon transform are presented. This talk is based on [1].

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COMPTON SCATTER TOMOGRAPHY: FROM ALGEBRAIC AND ITERATIVE RECONSTRUCTION TO MACHINE LEARNING

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Abstract

Introduction: X-ray and gamma-ray photons deposit energy and scatter when they interact with human tissue. Scattered radiation has historically been considered to be noise, contributing dose to the patient, but providing no useful information. Traditional imaging systems such as Computed Tomography (CT) and Positron Emission Tomography (PET) provide valuable diagnostic information but contribute to a growing population dose. With the increased availability of higher energy-resolution detectors and advanced Machine Learning approaches, scatter imaging is becoming feasible. This work will summarize the results of our CT and PET scatter-imaging research. I will illustrate the theory and will describe how the scatter-imaging techniques improve our ability to diagnose cancer without the dose or cost penalties associated with multi-modality scanning, can provide attenuation corrections, and give an anatomical context to PET, which is otherwise a functional imaging modality.

Methods and Materials: Our initial work in CT used a refined version of the backprojection algorithm over arcs to demonstrate that by measuring the energy and position of detected photons, we can reconstruct images with significantly fewer projections. For scatter-reconstruction in PET, generalized Maximum-Likelihood Expectation-Maximization (G*S-MLEM) reconstruction algorithms were developed, which are capable of reconstructing single and dual-scattered photons. The scattering angle, calculated from the energy of each photon using the Compton equation, was used to define two circular arcs (TCA) that encompass the annihilation position. In the zero-degree scattering angle limit, the scattered coincidences approach the true coincidence, and hence trues are considered a subset of scattered coincidences. To avoid overcorrecting for scattered coincidences, the attenuation coefficient was calculated by integrating the differential Klein-Nishina cross-section over a restricted-energy range, accounting only for scattered photons that were not detected. Phantoms containing cold and hot regions with various activities were simulated using the GATE platform. Energy resolutions of 5% to 20% were used to blur the simulated events, with scatter fractions from 10 to 40%. Images were reconstructed using different algorithms with a 350-650 keV energy window and the proposed restricted attenuation correction. Convolutional Neural Networks have been trained and tested using 120,000 PET images generated using simulated analytical PET images and with Monte Carlo simulation. The Monte Carlo simulation was used to obtain the energy distribution of the scattered photons, and transfer learning has been used to minimize the training data requirements.

Results and Conclusions: We have developed reconstruction algorithms capable of reconstructing CT and PET images using scattered photon coincidences. These approaches improve contrast, decreases noise, and eliminates the need for scatter corrections. Using scattered photons lowers the data required for equivalent image quality, reduces scan time, injected dose, and radiation burden. Images generated using single scattered coincidences demonstrate that only 70% of annihilation positions were correctly identified, while the generalized dual-scatter GDS-MLEM algorithm encompassed 98% of source positions. The GDS-MLEM has the highest sensitivity, improved contrast recovery coefficient, and reduced noise by 7.6% to 13.2% and 12.4% to 24.0%, respectively. The revised attenuation correction method facilitates dynamic selection of an energy window to provide optimum PET images. The GDS-MLEM approach is less sensitive to energy resolution and shows promise if detector energy resolutions of 12% can be achieved. While this algorithm is an improvement over our earlier approaches, our AI-based CNN deep-learning reconstruction approaches promise to provide still better quality images that are less sensitive to detector energy resolution.

FOURIER RECONSTRUCTION IN DIFFRACTION TOMOGRAPHY

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Abstract

We study the mathematical imaging problem of optical diffraction tomography (ODT) for the scenario of a rigid particle rotating in a trap created by acoustic or optical forces. Under the influence of the inhomogeneous forces, the particle carries out a time-dependent smooth, but irregular motion. The rotation axis is not fixed, but continuously undergoes some variations, and the rotation angles are not equally spaced, which is in contrast to standard tomographic reconstruction assumptions. In the present work, we assume that the time-dependent motion parameters are known, and that the particle's scattering potential is compatible with making the first order Born or Rytov approximation. By the Fourier diffraction theorem, the measurements of the scattered wave are related to the Fourier transform of the scattering potential on an irregular grid. We derive novel backpropagation formulae for the reconstruction of the scattering potential, which depends on the refractive index inside the object, taking its complicated motion into account. This provides the basis for solving the ODT problem with an efficient non-uniform discrete Fourier transform [2,3].

Furthermore, we consider the case of missing phase information, since often only intensity measurements are available in practice. We propose a new reconstruction approach [1] for ODT with unknown phase information, utilizing a hybrid input-output scheme with TV regularization. The so-obtained 2D and 3D reconstructions are even comparable to the ones with known phase.

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PERIODIC VIEW OF SOME CLASSICAL OPERATORS ASSOCIATED TO RADON TRANSFORMS

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Abstract

We discuss our recent results and ongoing work on the periodic Radon transforms. Certain dense partial data of the usual Radon transform can be mapped into periodic Radon transform data, which motivates studying the periodic counterpart of the Radon transform. Geometrically in two dimensions, the periodic Radon transform is the geodesic ray transform over closed geodesics of the flat torus. We compare some classical operators and results in both settings such as the Fourier slice theorem, the (filtered) backprojection, adjoint, normal and inverse operators, and their mapping properties. The periodic Radon transforms enjoy some remarkable properties which give simple reconstruction formulas and sometimes a bit surprising results that do not resemble their classical counterparts. One can obtain stability and regularization results in low regularity when data spaces are equipped with suitable (weighted) Sobolev or Bessel type structures.

This talk is based on series of works by Joonas Ilmavirta (Jyväskylä), Olli Koskela (Tampere), and the author; arXiv:1402.6209, arXiv:1906.05046, arXiv:1909.00495.

DIFFRACTIVE TENSOR FIELD TOMOGRAPHY AS AN INVERSE PROBLEM FOR A TRANSPORT EQUATION

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Abstract

We consider a general setting for dynamic tensor field tomography in an inhomogeneous refracting and absorbing medium as inverse source problem for the associated transport equation. Following Fermat's principle the Riemannian metric in the considered domain is generated by the refractive index of the medium. There is wealth of results for the inverse problem of recovering a tensor field from its longitudinal ray transform in a static euclidean setting, whereas there are only few inversion formulas and algorithms existing for general Riemannian metrics and time-dependent tensor fields. It is a well-known fact that tensor field tomography is equivalent to an inverse source problem for a transport equation where the ray transform serves as given boundary data. We prove that this result extends to the dynamic case. Interpreting dynamic tensor tomography as inverse source problem represents a holistic approach in this field. To guarantee that the forward mappings are well-defined, it is necessary to prove existence and uniqueness for the underlying transport equations. Unfortunately the bilinear forms of the associated weak formulations do not satisfy the coercivity condition. To this end we transfer to viscosity solutions and prove their unique existence in appropriate Sobolev (static case) and Sobolev-Bochner (dynamic case) spaces under a certain assumption that allows only small variations of the refractive index.

Numerical evidence is given that the viscosity solution solves the original transport equation if the viscosity term turns to zero

FROM ANALYSIS TO ITERATIVE ALGORITHMS: A LINEAR SYSTEMS PERSPECTIVE ON THE BROKEN RAY TRANSFORM

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Abstract

Analysis of the BRT has led to elegant inversion formulas for bounded images. A linear systems perspective provides a common framework for summarizing prior results and the challenges extending them to sampled data. We recognize two issues with the BRT: unbounded support of the data, and nontrivial nullspace of the forward operator. Addressing these challenges enables computationally efficient inversion formulas in the frequency domain for arbitrary ray directions. Further, our results yield fast implementations of the forward and backward transforms which are required for a broad class of iterative reconstruction algorithms. Recently we have used iterative algorithms to jointly estimate scatter density images and attenuation images from noisy data. Our results illustrate the sensitivity of joint image reconstruction to problem scaling and quantify the benefits of incorporating transmission measurements and additional scatter angles.

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A JOINT RECONSTRUCTION AND LAMBDA TOMOGRAPHY REGULARIZATION TECHNIQUE FOR ENERGY RESERVED X-RAY IMAGING

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Abstract

Here we present new joint reconstruction and regularization techniques inspired by ideas in microlocal analysis and lambda tomography, for the simultaneous reconstruction of the attenuation coeffcient and electron density from X-ray transmission (i.e., X-ray CT) and backscattered data (assumed to be primarily Compton scattered). To demonstrate our theory and reconstruction methods, we consider the "parallel line segment" acquisition geometry of Webber and Miller (Compton scattering tomography in translational geometries. *Inverse Problems,* 36, no. 2 (2020): 025007), which is motivated by system architectures currently under development for airport security screening.

We first present a novel microlocal analysis of the parallel line geometry which explains the nature of image artefacts when the attenuation coeffcient and electron density are reconstructed separately. We next introduce a new joint reconstruction scheme for low effective Z (atomic number) imaging ($Z < 20$) characterized by a regularization strategy whose structure is derived from lambda tomography principles and motivated directly by the microlocal analytic results.

Finally we show the effectiveness of our method in combating noise and image artefacts on simulated phantoms.

MINISYMPOSIUM
M7: Recent Advances in Inverse Problems of Time-Harmonic Wave Propagation

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Many developing and emerging fields in physics, astronomy and engineering are rich sources of inverse problems of time-harmonic wave propagation. The aim of this minisymposium is to bring together experienced and young researchers interested in time-harmonic inverse problems and to discuss recent advances, challenges and applications of this discipline. Among other things, we will focus on uniqueness, reconstruction algorithms, problems with partial data and simulations.

EIGENVALUE PROBLEM FOR A SCATTERER WITH A CONDUCTIVE BOUNDARY

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Abstract

In this talk, we will investigate the inverse acoustic scattering problem associated with an inhomogeneous media with a conductive boundary. We consider the corresponding classical transmission eigenvalue problem as well as the zero-index eigenvalue problem. This is a new class of eigenvalue problem that is not elliptic, not self-adjoint, and non-linear, which gives the possibility of complex eigenvalues.

We investigate the convergence of the eigenvalues as the conductivity parameter tends to zero as well as prove existence and discreteness for the case of an absorbing media.

UNIQUENESS RESULT FOR RANDOM INVERSE SOURCE PROBLEMS AND APPLICATIONS TO HELIOSEISMIC HOLOGRAPHY

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Abstract

We consider the problem of reconstructing the intensity of an uncorrelated random source from measurements of solutions to a Helmholtz-type equation at some distance to the source. We prove that under mild assumptions the intensity is uniquely determined by correlations of the random fields. In contrast, it is well that corresponding deterministic inverse source problems are highly non-unique, i.e. it is not possible to determine individual source realizations.

Such problems appear in locating noise source in aircrafts, trains or cars as well as in local helioseismology. We also discuss some well-established algorithms for reconstructing source intensities in a aeroacoustics and helioseismic holography as well as some new algorithms and show numerical results.

A DIRECT AND INVERSE SCATTERING PROBLEM FOR A LOCALLY PERTURBED PERIODIC STRUCTURE

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Abstract

In this talk we will study the scattering of time-harmonic point sources (or other sources of compact support) in some domain of the plane which is a local perturbation of a domain D where D is periodic with respect to the horizontal axis and bounded with respect to the vertical axis.

The main part of the talk is concerned with the discussion of a suitable radiation condition for this scattering problem and present results for uniqueness and existence. Then we turn to the inverse problem to determine properties of the perturbation from the measured fields and prove uniqueness of the inverse problem.

INTERIOR TRANSMISSION EIGENVALUES FOR FRACTAL DOMAINS

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Abstract

Interior transmission eigenvalues arise in the study of reconstruction algorithm for inverse scattering problems. The numerical calculation of those is a challenging task due to the fact that the problem is non-linear, non-elliptic, and non-self-adjoint (see for example [1] and [2]). It gets even more complicated when the two-dimensional domain under consideration has a fractal boundary. It is shown that we are able to successfully compute interior transmission eigenvalues for such domains.

Interesting observations and conjectures are made which hopefully stimulate the community to pursue further research in this direction.

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SINGLE MODE MULTI-FREQUENCY FACTORIZATION METHOD FOR THE INVERSE SOURCE PROBLEM IN ACOUSTIC WAVEGUIDES

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Abstract

This talk addresses the inverse source problem with a single propagating mode at multiple frequencies in an acoustic waveguide. The goal is to provide both theoretical justifications and efficient algorithms for imaging extended sources using the sampling methods. In contrast to the existing far/near field operator based on the integral over the space variable in the sampling methods, a multi-frequency far-field operator is introduced based on the integral over the frequency variable. This far-field operator is defined in a way to incorporate the possibly non-linear dispersion relation, a unique feature in waveguides. The factorization method is deployed to establish a rigorous characterization of the range support. A related factorization-based sampling method is also discussed. These sampling methods are shown to be capable of imaging the range support of the source.

Numerical examples are provided to illustrate the performance of the sampling methods, including an example to image a complete sound-soft block.

QUANTITATIVE PASSIVE IMAGING BY ITERATIVE HELIOSEISMIC HOLOGRAPHY

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Abstract

In local helioseismology one studies solar oscillations at the solar surface (e.g. line-of-sight velocities $\psi(\boldsymbol{r}, t)$) in order to learn about subsurface quantities (sound speed c, density ρ, flows u, wave attenuation γ) [1]. Due to the stochastic nature of solar oscillations the input data consists of cross-correlations $C(\boldsymbol{r}_1, \boldsymbol{r}_2, t)$ at any point pair $\boldsymbol{r}_1, \boldsymbol{r}_2$ at the solar surface. In the frequency domain the cross-correlation takes the form $C(\boldsymbol{r}_1, \boldsymbol{r}_2, \omega) = \psi(\boldsymbol{r}_1, \omega)\psi(\boldsymbol{r}_2, \omega)^*$. Acoustic wave propagation in the Sun can be described by a simplified scalar equation in the frequency domain

$$L\psi := -(\Delta + k^2)\psi - \frac{2i\omega}{\rho^{1/2}c}\rho\boldsymbol{u} \cdot \nabla\left(\frac{\psi}{\rho^{1/2}c}\right) = s, \quad k^2 = \frac{\omega^2 + 2i\omega\gamma}{c^2} - \rho^{1/2}\Delta\rho^{-1/2}, \qquad (1)$$

where s is the stochastic source term. Under the reasonable assumption of equipartition of the source power, the cross-covariance $E[C(\boldsymbol{r}_1, \boldsymbol{r}_2, \omega)]$ can be assumed to be proportional to $Im\big(G(\boldsymbol{r}_1, \boldsymbol{r}_2, \omega)\big)$ [2].

In this presentation, we will address the Inverse Problem of reconstructing $q \in \{c, \rho, \gamma, \boldsymbol{u}\}$ (in particular axisymmetric subsurface flows) from observations of cross-correlations at the solar surface. The cross-correlation data is a very large five-dimensional data set, which is unfeasible to store and can not be inverted directly. Therefore some a priori averaging in spatial or frequency direction is necessary. Traditional approaches reduce the cross-correlation to a smaller number of physically interpretable measurements (e.g. travel times) with acceptable signal-to-noise ratio. Helioseismic holography consists in first backpropagating the data and afterwards locally cross-correlating the backpropagated wavefield in order to image the solar interior at a previously defined target location. This way holography uses the whole cross-correlation data implicitly without the need to compute the cross-correlation explicitly. Nevertheless seismic holography only provides feature maps and is no quantitative regularization method. We will introduce an appropriate forward operator mapping to the cross-covariance at the surface [3] in order to solve the inverse problem using the Iteratively Regularized Gauss Newton Method with Conjugate Gradient inner iterations. The holographic backpropagation can be interpreted as the adjoint of the Frèchet derivative of the forward problem, such that the first iteration coincides with traditional holographic image intensity. This iterative approach is motivated by 3 main advantages compared to traditional approaches (like time-distance helioseismology):

1) The whole input cross-correlation data can be used without the loss of information caused by a priori reduction steps.
2) Helioseismology can be extended to nonlinear inverse problems, stepping forward to full-waveform inversions.
3) The gradient and the adjoint can be obtained by solving similar equations to (1) with modified right hand side, which can be implemented in a computational efficient way.

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FORMULAS FOR PHASE RECOVERING FROM PHASELESS SCATTERING DATA

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Abstract

We present formulas for phase recovering from appropriate monochromatic phaseless scattering data in dimension $d \geq 1$. In particular, these formulas reduce phaseless inverse scattering to standard inverse scattering with phase information.

This talk is based, in particular, on works [1-4].

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ON COMPLEX TRANSMISSION EIGENVALUES

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Abstract

Transmission eigenvalues arise in the study of inverse scattering problems for time-harmonic and penetrating waves. They are closely connected to non-scattering wavenumbers and depend on the refractive index of some practically unknown scattering domain which one aims to reconstruct. Sampling methods, which are designed for this purpose and exploit far field measurements of monochromatic incident waves, may fail correspondingly. On the other hand, given a fixed scattering domain such as a ball, it has been shown that the spectrum of all transmission eigenvalues does determine spherically stratified media uniquely under mild regularity assumptions, see [1].

Since the underlying transmission problem is a non-selfadjoint eigenproblem, it may also admit complex-valued eigenvalues in general. While numerical studies indicate their existence for quite general scatterers, see [2], theoretical justification has only been provided for the spherically stratified case so far, see [1]. In this talk we want to retackle this survey for homogeneous media and present new structural insights for complex-valued transmission eigenvalues of a disc/ball. Numerical experiments show that the concept may be extendible to other scattering shapes as well.

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MINISYMPOSIUM
M8: Inverse Problems in Science and Engineering

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Inverse problems arise in many areas of mathematical physics and applications are rapidly expanding to geophysics, chemistry, medicine and engineering. This minisymposium focuses on both analytical and computational methods for inverse problems in Science and Engineering. The approaches developed for such problems generally include numerical approximations, stability analysis, proofs of uniqueness and/or existence of the solution.

The minisymposium aims at bringing together well established scientists as well as young researchers working on inverse problems for partial differential equations. The topics of the minisymposium range from the mathematical modelling and the theoretical analysis of inverse problems for partial differential equations where some parameters (right-hand side, kernel, diffusion coefficient, etc.), unknown boundary condition(s) or portion of the boundary are to be found, to the development of efficient numerical schemes and their practical implementations.

ON 3D PET IN SPHERICAL GEOMETRY OF DATA ACQUISITION

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Abstract

The task of image reconstruction in positron emission tomography (PET) is mathematically equivalent to inversion of the X-ray transform from severely undersampled and truncated data. Several methods of dealing with these incomplete data problems have been developed throughout the years, including reprojection algorithms and various exact and approximate rebinning techniques. However, most of these methods have been created and optimized for the classical cylindrical geometry of data acquisition.

This talk is dedicated to the problem of image reconstruction in PET with a spherical geometry of data acquisition. We will discuss the major differences between the two geometries in relation to the existing methods of image reconstruction, present some new approaches to the solution of the problem and demonstrate the results of numerical studies.

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INVERSE PROBLEM FOR QUANTIFICATION OF LOCALIZED DAMAGE USING 1D NONLINEAR ELASTIC TWO-WAVE MIXING

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Abstract

Localized damage in a one-dimensional rod of length L is modeled as a quadratically nonlinear material of thickness a sandwiched between two identical linear materials. Approximate analytical solutions obtained by solving a one-dimensional nonlinear elastic wave equation indicate that the interaction of a single elastic wave with the nonlinear material generates higher harmonics. The damage level can be measured using the higher harmonic generation technique, but the quantification of the localized damage size has not been reported. Approximate analytical solution for $1D$ backscatter and transmitted waves from quadratically nonlinear material sandwiched between two linear materials has been obtained by Wang and Achenbach [1]. Considering the amplitudes of the higher harmonics of the backscattered and the transmitted waves an inverse problem is formulated to estimate damage size. The inverse problem is solved by reformulating it as a constrained optimization problem as follows:

$$\underset{a}{Minimize}\left|\frac{\sin\left(2a\omega_2 c^{-1}\right)}{2a\omega_2 c^{-1}}\right| - R_{2\omega_2 synth}\text{; Constraints: }\left|\frac{\sin\left(2a\omega_1 c^{-1}\right)}{2a\omega_1 c^{-1}}\right| = R_{2\omega_1 synth},\ a > 0 \quad (1)$$

where ω_1, ω_2 are angular frequencies and c is wave speed respectively.

Synthetic data ($R_{2\omega_2 synth}$, $R_{2\omega_1 synth}$) are generated for known damage sizes and is used to estimate the actual damage sizes by using the proposed formulation. It has been observed that this approach is limited up to a certain level of damage size (e.g. 0.1-1 mm damage size in aluminum at input frequencies of 1 and 1.5 MHz). To increase the range of the estimated damage size, the original approach has been improved by using the one way two-wave mixing method. Approximate analytical solution for the backscattered and transmitted waves due to two-wave mixing from quadratically nonlinear region is obtained and used to formulate a more robust inverse problem as follows:

$$\underset{a}{Minimize}\left|\frac{\sin\left(2a\omega_2 c^{-1}\right)}{2a\omega_2 c^{-1}}\right| - R_{2\omega_2 synth};$$

$$\text{Constraints: }\left|\frac{\sin\left(2a\omega_1 c^{-1}\right)}{2a\omega_1 c^{-1}}\right| = R_{2\omega_1 synth},\quad \left|\frac{\sin\left(2a(\omega_1-\omega_2)c^{-1}\right)}{2a(\omega_1-\omega_2)c^{-1}}\right| = R_{2\omega_{12}\,synth},\ , \qquad (2)$$

$$\left|\frac{\sin\left(2a(\omega_1+\omega_2)c^{-1}\right)}{2a(\omega_1+\omega_2)c^{-1}}\right| = R_{2\omega_{21}\,synth},,\ a > 0\ .$$

It can be observed that the additionally generated harmonics help to solve the inverse problem. Both the optimization problems (Equation (1) and Equation (2)) are solved numerically using gradient and non-gradient based algorithms to obtain the damage size. Using the improved approach, the detected damage size increases from 0.1-1.0 mm to 0.1-10 mm for the same range of input frequencies. In addition, only one mixing experiment is required in the second approach as compared to two separate experiments which need to be performed for the original approach.

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NUMERICAL SOLUTION OF DIRECT AND INVERSE PROBLEMS WITH PARAMETER UNCERTAINTY OF PHASE CHANGE ENERGY STORAGE WALL

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Abstract

In this paper, a numerical method based on quadtree scaled boundary element method and interval method is proposed to solve the forward and inverse heat transfer problems with uncertain parameters of phase change energy storage wall. The quadtree scaled boundary element method is used to analyze the phase change heat transfer directly based on the wall image, and the numerical model of the uncertain direct and inverse problems of phase change heat transfer is established by using the interval method. In the uncertainty analysis, the calculation of interval analysis can be significantly reduced by constructing Legendre polynomial surrogate model to approximately replace the calculation of deterministic problems. Numerical examples show that the proposed method can not only effectively evaluate the influence of the interval uncertainty of phase change wall material parameters on the temperature change, but also obtain the material design parameters through the prescribed temperature interval.

This work provides an effective numerical model for the analysis and design of heat transfer performance of phase change energy storage wall.

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IMAGE RECONSTRUCTION IN DIFFUSE OPTICAL TOMOGRAPHY: AN OPTIMAL BAYESIAN ESTIMATOR FOR ABSORPTION COEFFICIENT

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Abstract

In this talk, we will discuss an optimal Bayesian estimator including the rate of convergence to reconstruct the absorption coefficient for the Diffuse Optical Tomography (DOT). Mathematically, the reconstruction of the internal absorption or scattering coefficients is a severely ill-posed inverse problem and yields a poor quality image reconstruction.

We will present the efficacy of the proposed approach using simulations.

AN ITERATIVE METHOD FOR THE CAUCHY PROBLEM FOR THE HELMHOLTZ EQUATION

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Abstract

We consider the problem of reconstructing the acoustic or electromagnetic field from inexact data given only on an open part of the boundary of a given domain. The boundary is divided into two nonoverlapping parts. The model problem under study is the Cauchy problem for the Helmholtz equation and it is ill-posed in the sense of Hadamard. For the reconstruction, we first introduce an artificial auxiliary boundary in the interior of the domain and a positive parameter which are chosen to guarantee the positivity of a quadratic form associated to the Helmholtz operator, the introduced auxiliary boundary, and the chosen parameter as it has been discussed in [1].

We present an iterative method for which two well-posed boundary value problems are solved in every step. The algorithm works as follows: We initially choose Neumann data on an unknown part of the boundary and Robin-type jump conditions on the interior artificial boundary. For the first problem, we solve the boundary value problem for the Helmholtz equation where the specified Dirichlet boundary data together with the initially chosen Neumann data on the unknown part of the boundary and the Robin-type jump conditions on the artificial interior boundary are used. For the second problem, the boundary value problem for the Helmholtz equation is solved where the specified Neumann data is updated with Neumann data from the first solved problem and homogeneous Dirichlet data are considered on the other part of the boundary and on the interior artificial domain. We update the initially chosen Neumann data on the unknown part of the boundary with Neumann data from the previously solved problem. We also updated Robin-type jump conditions with similar conditions from the previously. We then solve the two problems described above.

The convergence of the proposed method will be provided. The numerical results performed using the finite difference method will be provided. Those results show that the convergence of the proposed iterative method require few number of iterations than the iterative algorithm presented in [1].

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STATISTICAL LINEAR INVERSE PROBLEMS BASED ON DISCRETELY SAMPLED FUNCTIONAL DATA

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Abstract

In this talk we investigate the performance of the spectral regularization methods when data is a collection of discretely sampled realizations of a process modelled as a classical statistical inverse problem. Minimax rates of convergence are established under both the common and the independent design. Different phase transition phenomena are studied leading to a partition of the functional data into three types (non-dense, dense and ultra-dense) in this framework.

Finally, the effect of the sampling frequency and the number of curves is illustrated in a numerical application.

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RECONSTRUCTING MOLECULAR FLEXIBILITY IN CRYOGENIC ELECTRON MICROSCOPY

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Abstract

Cryogenic electron microscopy (cryo-EM) is a powerful method to obtain the 3D structure of macromolecules from thousands of noisy projection images. Since these projection images emerge from many different observations of the same molecule, flexible areas of it result in a local drop of resolution in a single 3d reconstruction. We propose a method based on an variational autoencoder (VAE) that uses gaussian basis functions as a model for the molecule and estimate smooth deformation fields for every projection image. We further use the estimated deformations to better resolve the flexible regions in the reconstruction using a filtered backprojection algorithm along curved lines.

We present results on real data showing that we obtain improved 3D reconstruction.

REAL-TIME ELECTRICAL IMPEDANCE IMAGING AT HIGH AC FREQUENCIES

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Abstract

We investigate a multifrequency impedance imaging technique that has recently been suggested for mammography screenings. The technique uses boundary voltages generated by time harmonic (AC) boundary currents with either the same spatial distribution but different driving frequencies or different spatial distributions but same driving frequency. The proposed algorithms for the complex conductivity reconstructions are non-iterative and suitable for real-time imaging.

Numerical inversions obtained for two dimensional domains from simulated noisy data are presented.

This is joint work with Jeremy Curmi.

MINISYMPOSIUM
M9: Recent Advances in Analytical and Numerical Methods in Inverse Problems for Partial Differential Equations

Minisymposium dedicated to the 70th anniversary of Professor Michael V. Klibanov

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Many inverse problems of practical interest involve estimating unknown parameters from measurements of a state described by a partial differential equation (PDE) depending on these parameters; we mention just seismic inversion and acoustic medical imaging (which require determining the speed of sound in a wave equation) and nondestructive testing for corrosion (where an unknown coefficient in a Robin boundary condition has to be reconstructed). Such problems are challenging as they are usually nonlinear and require efficient large-scale methods for their numerical solution. In addition, analytical results are often crucial in developing appropriate numerical algorithms, since the function space structure of PDEs is an essential part of the problem. In this minisymposium talks by experts will be presented on recent developments in this area, including in particular those concerning global uniqueness and global convergence methods pioneered by Professor Klibanov.

BANG-BANG OPTIMAL CONTROL IN SPIN DYNAMICS OF RADICAL PAIRS IN QUANTUM BIOLOGY

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Abstract

Optimal control of the external electromagnetic fields and internal hyperfine parameters for the maximization of the quantum singlet-triplet yield of the radical pairs in biochemical reactions are analyzed. The system is modeled by Schrödinger equation with spin Hamiltonians given by the sum of Zeeman interaction and hyperfine coupling interaction terms. First- and second-order Fréchet differentiability in Hilbert spaces is proved and the formula for the first- and second-order Fréchet derivatives are derived. Pontryagin's maximum principle is proved and the band-bang structure of the optimal control is established. A closed optimality system for the identification of the bang-bang optimal control is revealed. It consists of system of nonlinear ODEs for finding the optimal state vector and its adjoint, and with subsequent identification of the bang-bang optimal control via Pontryagin's maximum principle. We hypothesize that the bang-bang optimal control and the corresponding optimal state vector is the quantum control theory counterpart of the quantum coherence concept, i.e. the ability of the system to exist in several distinct states simultaneously, such as the inter-system crossing between Singlet-Triplet states. Several numerical algorithms are implemented for solving nonlinear optimality system, and numerical results are demonstrated.

The results contribute towards understanding the structure-function relationship of a putative magnetoreceptor to manipulate and enhance quantum coherences at room temperature and leveraging biofidelic function to inspire novel quantum devices.

CARLEMAN BASED RECONSTRUCTION ALGORITHM

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Abstract

With Lucie Baudouin and Sylvain Ervedoza, we are interested in recovering coefficients in evolutionary partial differential equations. If uniqueness and stability results are generally well known, we have recently proposed the C-bRec (for Carleman based Reconstruction) algorithm to solve these inverse problems. Following an idea developed by Michael Klibanov, we use a Carleman estimate, a theoretical tool previously intended to prove the well-posedness of the inverse problem, in the construction of the numerical method. In particular, we show that the method is globally convergent, i.e. it converges to the coefficient to be recovered whatever the initial data, thus overcoming the drawbacks of least squares methods. The C-bRec method was initially presented in [1, 2] in the case of recovering a potential in the wave equation and generalized to the recovery of a source term in a reaction-diffusion equation ([3] with Muriel Boulakia and Erica Schwindt). In this talk, we present in detail the case of the recovery of the propagation speed in a wave equation ([4] with Axel Osses) from the measurement of the normal derivative of the solution on a part of the boundary.

We explain the challenges related to the numerical implementation of the algorithm and illustrate its efficiency on one and two dimensional examples.

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MULTIBAND REGULARIZATION OF A COEFFICIENT INVERSE PROBLEMFOR THE WAVE EQUATION

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Abstract

Our results demonstrate, that learned parts in the reconstruction procedure can increase image This talk is concerned with reconstructing the spatially variable wave speed in a scalar wave equation whose values are assumed to be taken pointwise from a known finite set. Such a property can be included in Tikhonov regularization by a so-called "multibang" regularization term that is nonsmooth but convex.

We discuss well-posedness as well as the numerical solution of the regularized problem.

SOLUTION OF A THREE-DIMENSIONAL INVERSE ELASTOGRAPHY PROBLEM FOR PARAMETRIC CLASSES OF INCLUSIONS

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Abstract

Elastography is a combination of methods of oncological examination of tissues in medicine, based on differences in the elastic properties of healthy and tumor tissues. For example, it is well known that the Young's modulus of tumor tissue is several times larger than the Young's modulus of healthy tissue. Usually, any of the methods of elastography can be divided into 3 stages of the study: (i) compression of tissues by external sources; (ii) measurement or calculating the displacement field of the investigated biological tissue; (iii) the calculation of the elastic properties of the tissue by solving the inverse problem.

In this talk, we consider a quasistatic approach to elastography, in which a three-dimensional inverse problem is solved for an isotropic linearly elastic body under small surface compressions. In this case, we have a system of stationary partial differential equations, describing the relationship between tissue displacements and elastic properties of the tissue, that is Young's modulus and Poisson's ratio. To solve the problem, a special algorithm is proposed and justified.

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MINISYMPOSIUM

M10: Computational Methods for Inverse Problems and Applications

Minisymposium dedicated to the 75th anniversary of a distinguished expert in the field of inverse problems Professor Anatoly Yagola

Organizers:
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In the last decades, techniques and methods of regularization have been enriched by new approaches including parallel computations and evolutionary algorithms. We want to bring together researchers who are investigating fundamental approaches and modern technique in different applications of inverse problems. In this minisymposium talks by experts will be presented on recent developments in this area, including in particular methods pioneered by Professor Yagola.

OPTIMIZATION APPROACH IN INVERSE PROBLEMS OF DESIGNING THERMAL CLOAKING DEVICES

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Abstract

Transformation optics approach proposed in [1] in 2006 has been widely applied when solving problems of manipulation of electromagnetic and acoustic fields. Then this method has been extended to the thermal, magnetic, electric and other fields [2, 3]. This approach allowed to design metamaterial devices for manipulating heat flows such as invisibility cloaks, illusion devices, invertors, concentrators, etc. [see 4].

It should be noted that the technical implementation of the solutions obtained in the cited articles is associated with significant difficulties. One way to overcome these shortcomings is to replace the original exact cloaking problem with the problem of approximate cloaking and apply the optimization method to solve inverse problems. Just the optimization method is used in this paper for solving inverse problems of designing thermal devices.

The paper consists of two parts. The first part contains theoretical analysis which includes the study of solvability of direct and inverse problems for the heat conduction model under consideration, the derivation of an optimality system that describes the necessary conditions for the extremum, and the establishment of important properties of optimal solutions that are used to develop efficient numerical algorithms. Based on analysis of optimality system we prove a number of new theorems concerning local uniqueness and stability of optimal solutions.

The second part includes describing efficient numerical algorithms of solving problems of designing heat cloaking and shielding devices and analysis of results of numerical experiments. One can read about some obtained results in [5-7].

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COLLOCATION-VARIATION APPROACHES TO NUMERICAL SOLUTION OF DIFFERENTIAL-ALGEBRAIC AND VOLTERRA EQUATIONS OF THE FIRST KIND

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Abstract

The report deals with numerical solution of quite complicated ill-posed problems: initial problem in differential-algebraic (DAEs) and Volterra equations of the first kind. A notion of index [1] and degree of ill-posedness [2], [3] characterize the complexity of the DAEs and Volterra equations, respectively.

It is known that many methods developed for ordinary differential equations and Volterra equations of the second kind can be unstable or inapplicable for the problems under consideration.

We propose new collocation-variation approach to numerical solution of these classes of equations and formulate conditions and rate of convergence. We investigate these methods for A- and L- stability.

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UNIFORM STABILITY OF RECOVERING THE DIRAC OPERATOR WITH AN INTEGRAL DELAY FROM THE SPECTRUM

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Abstract

Let $\{\lambda_n\}_{n\in\mathbb{Z}}$ be the spectrum of the boundary value problem $D = D(m_1, m_2)$ of the form

$$By'(x) + \int_0^x M(x-t)y(t)\,dt = \lambda y(x), \quad 0 < x < \pi, \quad y_1(0) = y_1(\pi) = 0, \qquad (1)$$

where λ is the spectral parameter and

$$B = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}, \quad y(x) = \begin{pmatrix} y_1(x) \\ y_2(x) \end{pmatrix}, \quad M(x) = \begin{pmatrix} m_1(x) & m_2(x) \\ -m_2(x) & m_1(x) \end{pmatrix},$$

while the functions $m_j(x)$ are complex-valued, and $(\pi - x)m_j(x) \in L_2(0,\pi)$, $j = 1,2$.

Inverse Problem 1. Given the spectrum $\{\lambda_n\}_{n\in\mathbb{Z}}$, find the functions $m_1(x)$ and $m_2(x)$.

In [1], the unique solvability of Inverse Problem 1 was established and its constructive solution was obtained. In particular, we proved that, for any complex sequence $\{\lambda_n\}_{n\in\mathbb{Z}}$ to be the spectrum of a boundary value problem D of the form (1), it is necessary and sufficient to have the asymptotics

$\lambda_n = n + \kappa_n, \quad \{\kappa_n\} \in l_2, \quad n \in \mathbb{Z}.$

In this note, we establish uniform stability of Inverse Problem 1, i.e. the following theorem holds.

Theorem 1. *For any $r > 0$, there exists $C_r > 0$ such that*

$$\left\|(\pi - x)(m_j - \tilde{m}_j)(x)\right\|_{L_2(0,\pi)} \le C_r \left\|\{\lambda_n - \tilde{\lambda}_n\}\right\|_{l_2}$$

as soon as $\|\{\lambda_n - n\}\|_{l_2} \le r$ and $\|\{\tilde{\lambda}_n - n\}\|_{l_2} \le r$, where $\{\tilde{\lambda}_n\}_{n\in\mathbb{Z}}$ is the spectrum of $D(\tilde{m}_1, \tilde{m}_2)$.

Let us note that uniform stability of inverse problems for classical Sturm–Liouville and Dirac operators was studied in [2–4]. Here, we use a different approach relying on the uniform stability of the so-called main nonlinear equation of the inverse problem [5] as well as on the uniform stability of recovering the characteristic determinant of the problem D from its zeros [6]. Finally, note that uniform stability of inverse problems for some scalar integro-differential operators was studied in [7, 8].

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INVERSE PROBLEM CHALLENGES IN PRACTICAL SUBSURFACE GEOENGINEERING APPLICATIONS

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Abstract

The problem of subsurface reservoir prediction is naturally inverse as it is base inferring unknown subsurface properties given the dynamic Earth response and geological prior information. Reservoir predictions are made using porous media flow models calibrated to past observations, referred to as to history matching. Several challenges associated with implementing practical history matching solutions for decision making are: i) vast subsurface uncertainty not limited only to non-Gaussian priors of the unknown porous media characteristics (porosity, permeability, stratigraphic features, structural fault properties) but also to the mathematical description of the spatial porous properties' distributions; ii) complex non-Gaussian behavior of the likelihood function, which in practical decision making has a multivariate nature; iii) difficulty in decoupling numerical errors, parameter uncertainty, model inadequacy and data discrepancy to make inference prediction; iv) accurate posterior inference given large computational cost of full physics modelling and the limitation of the surrogate proxies.

The presentation will go over real examples that demonstrate the importance of tackling the outlined challenge for practical decision making in subsurface resources development. In particular, the presentation will reflect up on:

i) the importance of informative priors to ensure geological realism of inverse modelling solutions;
ii) ii) the value of multi-objective optimization and its approximate posterior inference;
iii) iii) the impact of the inverse model solution quality in terms of likelihood and accurate estimation of the posterior for optimal development decisions.

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ON ACCELERATION OF LANDWEBER METHOD FOR ILL-POSED PROBLEMS

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Abstract

We consider linear illposed problem

$$Au = f, \quad A \in \mathcal{L}(H,F), \quad f \in \mathcal{R}(A),$$

where H, F are Hilbert spaces and range $\mathcal{R}(A)$ is non-closed, kernel $\mathcal{N}(A)$ is non-trivial. Only noisy data f^{δ} are available instead of exact data $f, \left\| f - f^{\delta} \right\| \le \delta$. The approximate solution may be found by the Landweber iteration method

$$u_{n+1} = u_n + \omega A^*(f^{\delta} - Au_n), \quad \omega \in (0, \frac{1}{\|A\|^2}), \quad n \in N_0. \qquad (1)$$

In this method and in many other analogous methods iterations are often stopped via discrepancy principle on first index $n = n(\delta)$, for which $\left\| Au_n - f^{\delta} \right\| \le b\delta, \quad b > 1$. Then method (1) has order optimal error estimate on the source-like solutions, but often needs huge number of iterations.

Articles [1,2] propose and analyze the Nesterov acceleration for Landweber method

$$u_{n+1} = z_n + \omega A^*(f^{\delta} - Az_n), \quad z_n = x_n + \alpha_n(u_n - u_{n-1}), \quad \alpha_n = (n-1)/(n+\beta), \quad \beta > -1. \qquad (2)$$

In [2] $\beta = 1$ is fixed.

Stopping iterations via the discrepancy principle, this method needs about square root of number of iterations, needed in method (1). Papers [1, 2] show that fixed $\beta = const$ in (2) leads to saturation effect in convergence rate.

We propose some alternative choices of α_n depending not only on k, but also on residual. One example is

$$\alpha_n = \frac{\theta_n}{\theta_{n-1}}(1 - \theta_{n-1}), \quad \theta_n = \min\left\{ \max\left[\frac{\theta_{n-1}}{1+\theta_{n-1}}, \frac{\rho_{n-1} - \rho_n}{2\rho_{n-1} - \rho_n}, 1 \right] \right\}, \quad \rho_n = \left\| Au_n - f^{\delta} \right\|.$$

Extensive numerical experiments show that for proposed choices of α_n the accuracy of the approximation is similar to the method (1) and number of iterations is similar to the method (2).

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CARLEMAN ESTIMATES FOR GLOBALLY CONVERGENT NUMERICAL METHODS FOR COEFFICIENT INVERSE PROBLEMS

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Abstract

Since the field of Inverse Problems is an applied one, it is insufficient just to prove some theorems. Rather it is necessary to develop reliable numerical methods. However, conventional numerical methods for Coefficient Inverse Problems (CIPs) are unreliable. The reason is that they are based on the minimization of least squares cost functionals. These functionals are nonconvex. Therefore, as a rule, they have many local minima and ravines. Since any minimization procedure can stop at any local minimum, which can be far from the true solution, then these methods are unreliable and unstable.

In the past several years Klibanov and his research team have successfully developed a radically new and very effective numerical method of solving CIPs. Furthermore, this method is verified on a variety of microwave experimental data. This is the so-called "convexification" method. In the convexification one constructs a globally strictly convex weighted Tikhonov-like functional. Therefore, the problem of local minima is avoided. Furthermore, the convexification generates globally convergent numerical methods. The key to this functional is the presence in it of the so-called Carleman Weight Function. This is the function which is involved as the weight in the Carleman estimate for the corresponding Partial Differential Operator.

The convexification will be presented for a broad variety of CIPs. Numerical results will also be presented for both computationally simulated and experimental data

ON DISCREPANCY PRINCIPLES FOR TIKHONOV REGULARISATION

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Abstract

The linear operator equation of the first kind: $Au = f$ with a bounded linear operator $A: U \to F$, between Hilbert spaces U and F, to which a generalised solution u_+ exists, when only an equation: $A_\eta u = f_\delta$ with inexact data (A_η, f_δ) (a bounded linear operator $A_\eta: U \to F$, $f_\delta \in F$, together with an estimate $\gamma \equiv (\eta, \delta)$ of error in data: $\|A - A_\eta\| \le \eta$, $\|f - f_\delta\| \le \delta$) is known, is the abstract form for linear ill-posed problems. It can be effectively solved by Tikhonov's regularisation method as a variational problem of seeking the unique vector $u_{\alpha\gamma}$ minimising the smoothing functional: $\|A_\eta u - f_\delta\|^2 + \alpha\|u\|^2$ with the regularisation parameter $\alpha > 0$ [1-6], that can be found as the solution of the linear operator equation of the second kind:

$$(A_\eta^* A_\eta + \alpha I)u_{\alpha\gamma} = A_\eta^* f_\delta. \qquad (1)$$

Particularly, the inexact data can be finite-dimensional approximations to (A, f), additionally perturbed as a result of preceding measurements or computations; the estimates (η, δ) should then include all-type errors in data. A question of both theoretical and practical importance is the choice of α enabling the convergence $u_{\alpha\gamma} \underset{\gamma \to 0}{\to} u_+$. Effective rules for choosing the regularisation parameter are usually defined in the form of additional equation (2) to be satisfied by pair $(u_{\alpha_*\gamma}, \alpha_*)$ (together with (1)). The most usefull are the discrepancy principle [1,2,3], especially the generalised discrepancy principle [2]:

$$\|A_\eta u_{\alpha\gamma} - f_\delta\|^2 = (\eta\|u_{\alpha\gamma}\| + \delta)^2 + [\mu_\gamma^\kappa(f_\delta, A_\eta)]^2; \quad \mu_\gamma^\kappa(f_\delta, A_\eta) \ge \inf\{\|A_\eta u - f_\delta\|: u \in U\}; \qquad (2a)$$

or the second discrepancy principle [4]:

$$\alpha^q\|A_\eta^* A_\eta u_{\alpha\gamma} - A_\eta^* f_\delta\| = \beta\|A_\eta\|(\|u_{\alpha\gamma}\|\eta + \delta); \quad \beta > 0, \quad q > -\tfrac{1}{2}. \qquad (2b)$$

Another rule can be formulated as a combination of the Tikhonov regularisation and the method of quasi-solutions [5].

All regularisation algorithms with a-posteriori parameter choice rules include iterative processes for solving non-linear equations like (2a) or (2b) together with (1), combined with discretisation of equations (1) and (2). These algorithms are different to those of iterative regularisation constructed immediately for equation (1) [6]. The properties of methods with discrepancy principles and especially some new results for second-discrepancy principle will be considered and commented in the talk.

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HIGH-PERFORMANCE REGULARIZATION OF MULTI-PARAMETRIC INVERSE PROBLEMS OF EPIDEMIOLOGY AND SOCIAL NETWORKS

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Abstract

Mathematical models in natural sciences based on mass balance law and diffusive processes are described by systems of ordinary differential equations (ODE) and partial differential equations of parabolic type (PDE). Considered mathematical models are driven by a lot of parameters such as coefficients of ODE/PDE, initial conditions, source, etc., that play a key role in prediction properties of models. The most of parameters is unknown or can be rough estimated. The inverse problem consists in identification of parameters of mathematical models using additional measurements of some direct problem statements in fixed times. Considered inverse problems are ill-posed, i.e. its solutions could be non-unique and unstable to data errors. The identifiability and sensitivity analysis are used to construct the regularization method for solving of inverse problems [1].

Inverse problems are formulated as minimization problems of loss function depends on vector of parameters. To find the global minima of minimization problems the combination of machine learning (ML), heuristic (simulated annealing, particle swarm optimization, genetic algorithm, *etc*.) and deterministic approaches are implemented. The ML methods such as artificial neural network, support vector machine, etc., as well as heuristic ones identify the global minima domain, could be easily paralleled and do not use the loss function features. Then the gradient deterministic methods identify the global minima in its area with guaranteed accuracy [2, 3]. The continuous and discrete gradient of loss function are derived with its convergence estimates [4]. On the other hand, the loss function can be represented as a multi-scale tensor to which the tensor train (TT) decomposition can be applied. TT method is easily paralleled and uses the structure of the loss function [5]. The confidence intervals for control an accuracy of approximate inverse problem solution are constructed and analyzed [3].

The numerical results for solving of inverse problems for mathematical models of epidemiology and social processes in online social networks are presented and discussed.

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REGULARIZING METHODS FOR PREDICTING BULKY MOLECULES VIBRATIONAL SPECTRA BASED ON THE COMBINED USE OF *AB INITIO* AND EXPERIMENTAL DATA

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Abstract

A number of different mathematical problems arise in the data processing of experimental data obtained by means of infrared and Raman spectroscopy. The most important is the so-called inverse vibrational problem of determining parameters of the molecular force field (force constants) from given experimental data (vibrational frequencies, isotope frequency shifts, Coriolis constants, centrifugal distortion constants, etc.). The accumulation of data on force constants is necessary for prediction of spectra and other properties of the bulky compounds not yet investigated which are too large for acuurate quantum mechanical calculations.

The mathematical problem of calculating molecular force fields within the general approximation of small vibrations (harmonic model) is an ill-posed problem and it does not satisfy any of the three well-posedness conditions (existence of solution, its uniqueness and stability to perturbations in input data). In most cases, the main difficulty is non-uniqueness of solution. Different algorithms based on the theory of regularization of nonlinear ill-posed problems have been proposed for solviing this problem and finfing the sets of force constants [1]. In our strategy the stabilizing matrix F^0 is chosen as a result of quantum mechanical calculations, and thus we search for matrix F which is the nearest by the chosen Euclidean norm to the given *ab initio* F^0. The optimized solution is referred to as *Regularized Quantum Mechanical Force Field* (RQMFF). New numerical algorithm for the calculation of scale factors for the molecular force fields expressed in Cartesian coordinates is developed and implemented as a part of the software package SPECTRUM [2,3].

The importance of large molecular systems (biological objects, polymers, giant aggregates etc.) stimulates the development of special approaches for the describing their physicochemical properties such as molecule geometry, vibrational frequencies, and thermodynamic functions, etc. Fast growing computational resources and numerical methods lead to the great advantage of modern methods of quantum chemistry for the solving many problems of structural chemistry in application to the large molecular systems. But the accurate calculations of the large molecular systems consisting of a few hundred atoms are still limited by the dimensions of systems because the purely *ab initio* methods require very large computtaional resources. One of possible approaches in such cases is to compose the matrix of force constants of bulky molecules from separate fragments corresponded to the smaller size units of force constants calculated as the regularized quantum mechanical force field (RQMFF) or regularized scaled quantum mechanical force field (RSQMFF). Results obtained within this approach are demonstrated for some melatonin and pyridoxale derivatives.

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LOCAL SOLVABILITY AND STABILITY OF INVERSE SPECTRAL PROBLEM FOR NON-SELF-ADJOINT STURM-LIOUVILLE OPERATOR

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Abstract

We study the inverse problem of recovering a complex-valued square-integrable potential in the Sturm–Liouville equation from spectra of two boundary value problems $L_j(q)$ with one common boundary condition:

$$-y''(x)+q(x)y(x)=\lambda y(x), \quad x\in(0,\pi),$$

$$y(0)=y^{(j)}(\pi), \quad j=0,1.$$

Let $\lambda_{nj}\ {}_{n=1}^{\infty}$ be the spectrum of $L_j(q),\ j=0,1$ It is known that the potential is uniquely determined by these two spectra. In the self-adjoint case (when $q(x)$ is real-valued), the inverse problem of recovering the potential q from two spectra is well-studied. Borg was first who studied this inverse problem [1]. He has proved its local solvability and stability of the solution. In the self-adjoint case, there was also obtained the global solvability [2], which includes necessary and sufficient conditions on any sequences to be the spectra of some boundary problems $L_j(q)$. This result was generalized in [3] to the potentials from the continuous scale of Sobolev spaces, and uniform stability has been obtained under some assumptions.

For the complex-valued potentials, the analogs of classical Borg's results for $q\in L_2(0,\pi)$ were proved in [4]. The difficulty which arises there is that the multiple values in the spectra can occur. To overcome it, we generalised classical Borg's method to the case of multiple spectra. Using this approach, we proved solvability and stability of the inverse problem for complex-valued smooth potentials:

Theorem 1. For any complex-valued model potential $q\in W_2^1(0,\pi)$ there exists $\delta>0$ such that if arbitrary sequences $\tilde{\lambda}_{nj}\ {}_{n=1}^{\infty},\ j=0,1$ satisfy the condition

$$\Lambda:=\sqrt{\sum_{n=1}^{\infty} n^2\ \left|\lambda_{n0}-\tilde{\lambda}_{n0}\right|^2+\left|\lambda_{n1}-\tilde{\lambda}_{n1}\right|^2}<\delta,$$

then there exists a unique function $\tilde{q}\in W_2^1(0,\pi)$ such that $\tilde{\lambda}_{nj}\ {}_{n=1}^{\infty}$ are the spectra of the boundary value problems $L_j(\tilde{q}),\ j=0,1$ correspondingly. Moreover, $\|q-\tilde{q}\|_{W_2^1(0,\pi)}\le C\Lambda$, where the constant C depends only on q.

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FAST COMPUTATION OF TSUNAMI WAVE PARAMETERS BY MEASURED DATA INVERSION

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Abstract

Speeding up calculation of tsunami wave propagation is very important for the events in a near field zone. This is just the case of Japan, where it takes about 20 minutes after the quake for the wave to approach the nearest coast. Use of the field programmable gates array (FPGA) microchip makes it possible to achieve valuable performance gain with the modern regular personal computer. With the help of such a tool, we study the distribution of tsunami wave maximal heights along the coast at Southern part of Japan depending on particular location of tsunami source. Synthetic source covers area of 100x200 km rectangle and has realistic shape. As is observed numerically, only selected parts of the entire coast line are subject of dangerous tsunami wave amplitude. Particular locations of such areas strongly depend on location of tsunami source. It is possible to provide local authorities with the PC based software/hardware tool for fast (say, in a few minutes) evaluation of tsunami danger for particular village or industrial unit at the coast [1,2].

Source parameters of tsunami waves are an essential part of any modern tsunami warning system. Recalculation of a measured time series (wave profile obtained by a seabed based pressure sensor) in terms of initial sea surface displacement at tsunami source is among the most (or) one of the promising approaches to be applied in a warning center. The "orthogonal decomposition", that was proposed and studied earlier by the authors, is numerically studied here. Realistic shape of sea surface displacement and digital bathymetry of the southern part of Japan are used. To study functionality of the proposed approach, wave profiles are obtained at positions of the sensors of DONET – Dense Oceanfloor Network system for Earthquakes and Tsunamis – pressure gauge network of Japan. We stress on the quality of tsunami source parameters reconstruction as well as on time required. As observed, just a part of the first wave period is enough for robust determination of such source parameters as amplitude and total volume of water.

Results of numerical tests are summarized in table and then discussed [3,4].

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ASIMPTOTIC SOLUTIONS OF INVERSE COEFFICIENT PROBLEMS FOR BURGERS TYPE EQUATIONS WITH INTERIOR LAYERS

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Abstract

The possibility of applying asymptotic analysis methods to solving inverse problems for a nonlinear singularly perturbed Burgers-type equation (reaction-diffusion-advection equation) with time-periodic coefficients is shown. In particular, the problems of reconstructing the linear gain (reaction coefficient) describing the properties of the medium and the boundary conditions from known information about the observed solution of the direct problem over a certain time interval (period) are considered. Similar problems arise in gas dynamics, in nonlinear wave theory, biophysics, chemical kinetics, and many others practical applications and are described by non-linear parabolic equations with small parameters in the derivatives (see, for example, [1] - [4], and references therein). These problems are intensively studied in connection with the fact that they act as mathematical models that reveal the main mechanisms that determine the behavior and more complex models of nonlinear wave theory.

Recent results for some classes of initial boundary value problem for some classes of Burgers-type equations, for which we investigate moving fronts by using the developed comparison technique are presented. These results were illustrated by the problem

$$\varepsilon \frac{\partial^2 u}{\partial x^2} - A(u,x,t)\frac{\partial u}{\partial x} - \frac{\partial u}{\partial t} = (u,x,t), \quad x \in (0,1), t \in R,$$
$$u(0,t,\varepsilon) = u^0(t), \quad u(1,t,\varepsilon) = u^1(t), t \in R,$$
$$u(x,t,\varepsilon) = u(x,t+T,\varepsilon) \quad x \in [0,1], t \in R,$$

The work demonstrates further development asymptotic-numerical methods for solving direct and inverse problems with boundary and inner layers (see also [5], [6]. This approach is applied to a new class of time-periodic reaction-diffusion-advection problems with inner transition layers. The concept of an asymptotic solution of inverse coe_cient problems is introduced. This approach is demonstrated on the inverse coe_cient problem. For this equations the problem is reduced to a much simpler problem. This allows us to classify the original problem as a well-posed or ill-posed inverse problem. The proposed approach can be applied to sufficiently wide class of problems with boundary and inner layers.

This work was supported by the Russian Science Foundation (project N18-11-00042).

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FINITE DIFFERENCE REGULARIZED METHOD FOR SOLVING THE GENERALIZED ONE-DIMENSIONAL INVERSE PROBLEM OF PROPAGATION OF THE ACTION POTENTIAL ALONG NERVOUS FIBER

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Abstract.

We study the process of propagation of a nerve impulse along a nerve fiber in a one-dimensional case with an instantaneous source and a flat boundary. The process is described by a parabolic equation problem and, using the Laplace transform, is reduced to a hyperbolic equation problem. The problem of recovering the unknown coefficients of the equation is considered.

Using the characteristic method and the singularity extraction method, the inverse hyperbolic problem is reduced to a regular inverse problem with data on characteristics. A finite-difference method was applied to the last problem and the solution of the inverse problem was regularized.

A finite-difference regularized solution of an unknown coefficient is obtained and it is a finite-difference regularized solution of an inverse problem of parabolic type.

The convergence estimate is obtained and the convergence of the regularized solution of the inverse problem to the exact solution of the inverse problem is shown.

Formulation of the problem: The process of propagation of a nerve impulse along a nerve fiber in a one-dimensional case is described by a telegraph equation of a parabolic form [1]:

$$C_m(x)\frac{\partial u(x,t)}{\partial t} = \frac{r_a(x)}{2\rho_a(x)}\frac{\partial^2 u(x,t)}{\partial x^2} - \frac{u(x,t)}{\rho_m(x)\cdot l}, \quad x \in R_+,\ t \in R_+, \tag{1}$$

where $u(x,t)$ – is the intracellular action potential, $\rho_m(x)$, $\rho_a(x)$ – is the resistivity of the plasma (membrane) and nerve fiber, $r_a(x)$ – is the radius of the nerve fiber, $C_m(x)$– is the capacity per unit area of the membrane, l – is the thickness of the membrane, a, m – are the axons (nerve fiber) and membrane indices.

The inverse parabolic problem consists in determining one of the parameters $r_a(x)/\rho_a(x)/\rho_m(x)/C_m(x)$ with the remaining parameters known and for the given initial and boundary conditions of the form:

$$u(x,t)\big|_{t<0} \equiv 0, \quad u_x(x,t)\big|_{x=0} = h_0\theta(t) + r_0\theta_1(t) + p_0\theta_2(t), \quad t \in R_+, \tag{2}$$

h_0, r_0, p_0– positively constant numbers, $\theta(t)$– is the Heaviside theta function, $\theta_1(t) = t\theta(t)$, $\theta_2(t) = \frac{t^2}{2}\theta(t)$, as well as given additional information in the form:

$$u(x,t)\big|_{x=0} = g(t),\ t \in [0,2T], \tag{3}$$

where T – time, is a positively constant number. Here the / sign means or, i.e. one of the coefficients is determined with the remaining coefficients known.

We use the Laplace transform [2] to solve problem (1) - (3), that is,

$u(x,t) = \int_0^\infty V_t(x,\tau)G(t,\tau)d\tau = \int_0^\infty V(x,\tau)G_{tt}(t,\tau)d\tau$, where $G(t,\tau) = \frac{1}{\sqrt{\pi t}}e^{\frac{\tau^2}{4t}}$ is the Green's function, inverse problem (1) - (3) is reduced to a hyperbolic inverse problem and this inverse problem was solved by the finite-difference regularized method.

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INVERSE PROBLEMS IN THIN FILM OPTICS: RECENT ACHIEVEMENTS AND THE NEWEST TRENDS

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Abstract

Thin film optical coatings can be found in virtually all modern scientific instruments and consumer products. Examples range from high quality coatings for LIGO interferometer mirrors, space telescopes, nuclear fusion facilities, to bulk coatings for mobile phones, eyeglasses, architectural glass, and more. Coating technology has made tremendous progress over the past decades, and this is largely due to the progress in solving inverse problems in thin film optics. Two main classes of these problems are distinguished: synthesis problems associated with the design of coatings with given spectral characteristics, and a wide class of inverse reconstruction problems associated with various aspects of the practical production of optical coatings. Modern coatings can have many dozens and even hundreds of layers. The thicknesses of these layers are the main design parameters of optical coatings that determine their spectral characteristics. Due to the large number of design parameters, designing of an optical coating is an extremely difficult inverse synthesis problem; however, modern design methods make it possible to design coatings of any type [1, 2]. Moreover, in many cases, multiple designs can be obtained, potentially allowing the selection of the design with the best feasibility properties. The choice of such a design is closely related to the choice of a monitoring procedure for controlling the coating layer thicknesses during the production process. At present, various optical monitoring techniques are widely used to control the production of the most complex optical coatings [3]. Optical monitoring procedures are based on measuring the reflectance or transmittance of a coating during its deposition, either at one monitoring wavelength (monochromatic monitoring) or at a dense wavelength grid in a certain spectral region (broadband monitoring). The practical implementation of these procedures requires the solution of multiparametric inverse reconstruction problems. In the last few years, a new class of algorithms, called non-local algorithms, has been proposed to solve these problems [4, 5]. A distinctive feature of the non-local algorithms is that they utilize a huge amount of all the experimental data accumulated during the coating deposition process. They also make extensive use of the basic ideas of the regularization theory related to the stabilization of the solution to the inverse problem. As a result, the accuracy of solving inverse reconstruction problems has noticeably improved. Another direction in the production of high-quality optical coatings is the choice of the optimal combination of the theoretical coating design and the monitoring procedure used for its practical implementation. The theoretical basis for optimizing this choice is provided by recent results on the correlation of errors caused by different optical monitoring procedures and the error self-compensation effect associated with this correlation [6-8]. A universal approach to the study of the error correlation has been developed and an estimate for the strength of the error self-compensation effect has been proposed. Using this estimate, the combination of design and monitoring procedure that provides the best feasibility properties is selected.

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ON THE GENERALIZED ASYMPTOTICAL REGULARIZATION FOR LINEAR ILL-POSED PROBLEMS

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Abstract

In this talk, two new classes of iterative regularization methods for linear ill-posed operator equations will be presented: the damped second-order asymptotical regularization methods [1-5] and the fractional-order asymptotical regularization method [6]. Convergence rate under range type source conditions will be discussed. Saturation and converse results are given. We will also prove the acceleration phenomena of both methods. Several numerical examples are presented to show the effectiveness of the proposed new approaches.

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MACHINE LEARNING REGULARIZATION OF INVERSE PROBLEM FOR BLACK-SCHOLES EQUATION

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Abstract

The solution of the Black-Scholes equation [1] provides an estimate of the theoretical price of the option which is a derivative of some underlying asset (for example, stocks). The Black-Scholes model is a multi-parameter equation depending on the current price of the underlying asset s that is assumed to follow the geometric diffusion with Brownian motion W_t

$$ds_t = \mu s_t\, dt + \sigma s_t\, dW_t,$$

the strike price K of the option, the time to maturity of the option T expressed in years, the risk free rate r as well as the price volatility $\sigma(s)$ of the underlying asset expressed in years. We consider the Black-Scholes model in which the put option price function $u(s,T)$ with the payoff function $f(s) = \max\{s-K,0\}$, where $K>0$, is found as the solution to the direct problem:

$$\frac{\partial u}{\partial t} = s\,r\frac{\partial u}{\partial s} + \frac{1}{2}s^2\sigma^2(s)\frac{\partial^2 u}{\partial s^2} - ru, \quad s\in[0,\infty),\ t\in(0,T); \qquad (1)$$
$$u(s,T) = f(s),\quad u(0,t) = K.$$

Volatility is an important value of the underlying asset risk measure used by portfolio managers. However, it is not possible to clearly observe the volatility of the underlying asset of the option in the financial markets. In practice, market participants work with the so-called implied volatility $\sigma(s)$ from the problem (1) which is calculated based on the current value of the option observed on the market assuming that this value reflects the expected risks. The inverse problem for Black-Scholes equation (1) consists in identification of implied volatility using measured data of option price $g(t)$ at fixed times on the given underlying asset price curve $s=\varphi(t)$:

$$u(\varphi(t),t) = g(t), \quad t\in(0,T). \qquad (2)$$

The inverse problem (1) - (2) is ill-posed [2, 3] and reduced to the minimization least square problem that is solved by neural networks with gradient optimization approach [4]. The comparison analysis with other optimization methods is presented as well as the short-term prediction of option price behavior.

The work is supported by the Russian Science Foundation (project No. 18-71-10044) and by the Russian Foundation for Basic Research (project No. 18-31-20019).

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MINISYMPOSIUM
M11: Inverse Problems for Time-fractional PDEs

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Within years, partial differential equations with fractional derivatives have become very popular among researchers, owing it not only to their effectiveness in applied scientific modeling but also to their novel mathematical features.

This mini-symposium is mainly concerned with the mathematical analysis of inverse source problems and coefficient inverse problems for fractional-order partial differential equations, and it aims for bringing researchers together to present and discuss their latest achievements on these topics.

INVERSE SOURCE PROBLEMS FOR TIME-FRACTIONAL DIFFUSION (WAVE) EQUATIONS

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Abstract

Within the last decade, evolution equations with fractional derivatives have gathered considerable attention among both theoretical and applied disciplines due to their feasibility in modeling physical processes such as anomalous diffusion. In this talk, we consider initial-boundary value problems for time-fractional diffusion(-wave) equations represented by

$$(\partial_t^\alpha + L)u(\boldsymbol{x}, t) = \rho(t)g(\boldsymbol{x}), \quad \boldsymbol{x} \in \Omega, 0 < t < T,$$

where ∂_t^α $(0 < \alpha \leq 2)$ denotes the Caputo derivative and L is a symmetric elliptic operator. In this talk, we investigate the following two inverse source problems.

1. Fix $\boldsymbol{x}_0 \in \Omega$. Provided that g is known, determine ρ by the single point observation of u at $\{\boldsymbol{x}_0\} \times (0, T)$.
2. Fix a nonempty subdomain $\omega \subset \Omega$. Provided that ρ is known, determine g by the partial interior observation of u in $\omega \times (0, T)$.

The uniqueness results of 1 and 2 were first established in [5] and [1] based on the strong positivity property and the weak vanishing property of the operator $\partial_t^\alpha + L$, respectively. There are also researches on the corresponding numerical reconstructions and we refer to [3] as a comprehensive survey on the above inverse source problems before 2019.

In recent years, we further investigate Problem 1 and generalize the formulation on the following two directions:

(1) reducing the observation data as much as possible, and

(2) relaxing the observation period to an arbitrary time interval

The direction (1) is interesting from both theoretical and practical aspects. Meanwhile, the direction (2) provides great freedom in performing observation, which is essentially important in unexpected accidents. The second part of this talk is devoted to the brief explanation of the latest progress concerning the uniqueness on the above directions as well as the key ingredients of their proofs (see [2,4]).

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A UNIFIED FRAMEWORK FOR THE REGULARIZATION OF FINAL VALUE TIME-FRACTIONAL DIFFUSION EQUATION

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Abstract

My talk focuses on the regularization of backward time-fractional diffusion problem on unbounded domain. This problem is well-known to be ill-posed, whence the need of a regularization method in order to recover stable approximate solution. For the problem under consideration, I present a unified framework of regularization which covers some techniques such as Fourier regularization [3], mollification [2] and approximate-inverse [1]. I investigate a regularization technique with two major advantages: the simplicity of computation of the regularized solution and the avoid of truncation of high frequency components (so as to avoid undesirable oscillation on the resulting approximate-solution). Under classical Sobolev-smoothness conditions, I derive order-optimal error estimates between the approximate solution and the exact solution in the case where both the data and the model are only approximately known. In addition, an order-optimal a-posteriori parameter choice rule based on the Morozov principle is given. Finally, via some numerical experiments in two-dimensional space, I illustrate the efficiency of our regularization approach and we numerically confirm the theoretical convergence rates established in the paper.

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INVERSE PROBLEMS FOR FRACTIONAL DIFFUSION EQUATION WITH ONE MEASUREMENT

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Abstract

We consider an inverse problem for the space-time fractional diffusion equation

$$\left(\partial_t^\alpha + \left(-\Delta_g\right)^\beta\right) u(x,t) = f(x,t), \quad x \in M, t > 0,$$

where (M, g) is a complete Riemannian manifold without boundary, ∂_t^α is the Caputo derivative of order $\alpha \in (0,1]$, Δ_g is the Laplace-Beltrami operator on M, and $\beta \in (0,1]$. We show that it is possible to construct a source f that is supported on $V \times (0,T)$, where V is a given open subset of the manifold and $T > 0$, so that the manifold (M, g) is determined up to a Riemannian isometry by the evolution of the corresponding solution u on $V \times (0,T)$.

This is joint work with Tapio Helin, Matti Lassas, Reed Meyerson, and Zhidong Zhang.

NUMERICAL ESTIMATION OF A DIFFUSION COEFFICIENT IN SUBDIFFUSION

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Abstract

In this work, we consider the numerical recovery of a spatially dependent diffusion coeffcient in a subdiffusion model from distributed observations. The subdiffusion model involves a Caputo fractional derivative of order $\alpha \in (0,1)$ in time. The numerical estimation is based on the regularized output least-squares formulation, with an $H^1(\Omega)$ penalty. We prove the well-posedness of the continuous formulation, e.g., existence and stability.

Next, we develop a fully discrete scheme based on the Galerkin finite element method in space and backward Euler convolution quadrature in time. We prove the subsequential convergence of the sequence of discrete solutions to a solution of the continuous problem as the discretization parameters (mesh size and time step size) tend to zero. Further, under an additional regularity condition on the exact coeffcient, we derive convergence rates in a weighted norm for the discrete approximations to the exact coeffcient. The analysis relies heavily on suitable nonstandard nonsmooth data error estimates for the direct problem.

We provide illustrative numerical results to support the theoretical study.

MINISYMPOSIUM
M12: Electrical Impedance Tomography: Theory and Applications

Organizers:
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Electrical impedance tomography (EIT) is an imaging modality that consists in the determination of the electrical conductivity distribution inside a body from current and voltage measurements on its boundary. Applications include medical imaging, nondestructive testing and geophysical prospecting. Its mathematical formulation was proposed by A.P. Calderon in 1980 and it has triggered a huge amount of research since then. On the theoretical side, the main issue has been to prove uniqueness of the related inverse problem, namely, the injectivity of the measurement or forward map. Concerning applications, EIT faces major numerical hurdles, since the problem is severely ill-posed. In order to mitigate this instability, strategies ranging from regularization methods to compressed sensing and machine learning have been employed.

In this minisymposium, we are gathering experts of EIT and of the Calderon's problem to share recent theoretical and numerical/applied insights.

LIPSCHITZ STABLE DETERMINATION OF POLYGONAL AND POLYHEDRAL CONDUCTIVITY INCLUSIONS FROM BOUNDARY MEASUREMENTS

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Abstract

We will present some recent results obtained in collaboration with Elisa Francini and Sergio Vessella concerning global Lipschitz stability estimates for the Hausdorff distance of polygonal and polyhedral conductivity inclusions in terms of the Dirichlet-to-Neumann map

MACHINE LEARNING APPROACH FOR STROKE DETECTION IN ELECTRICAL IMPEDANCE TOMOGRAPHY

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Abstract

Reconstruction of images in electrical impedance tomography (EIT) requires the solution of a nonlinear inverse problem on noisy data. This problem is typically ill-conditioned and solution algorithms need either simplifying assumptions or regularization based on a priori knowledge. This work considers a different strategy, it aims at training some classification learners (SVM and Neural Networks) with absolute EIT measurements on a head model [1], in order to binary classify the type of stroke occurred: haemorrhagic, if there is a blood leak in the brain tissue, or ischaemic, when a blood clot stops the blood supply to a certain area of the brain. The computed measurements take into account different kinds of possible errors in the measurement setup: slight variations in the background conductivity and in the contact impedances, misplaced electrodes and mismodeled head shape.

Classification learners have been trained both in the basic case of one single layer of conductivity (meaning only brain conductivity taken into account) and with the insertion of a skull and a skin layer (respectively less than and as conductive as the brain tissue).

Results show reasonably high probability of detecting the correct type of stroke.

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THE CALDERÓN PROBLEM WITH LIPSCHITZ CONDUCTIVITIES

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Abstract

We will discuss the Calderón problem for Lipschitz conductivities. In particular, we will recall the uniqueness question, and we will discuss the difficulties to produce a reconstruction algorithm.

References

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GLOBAL CONVERGENCE AND STABLE INVERTIBILITY FOR A ROBIN TRANSMISSION PROBLEM WITH FINITELY MANY MEASUREMENTS

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Abstract

We derive a simple criterion that ensures uniqueness, Lipschitz stability and global convergence of Newton's method for finite dimensional inverse problems with a continuously differentiable, pointwise convex and monotonous forward function. Our criterion merely requires evaluating the directional derivative of the forward function at finitely many evaluation points and finitely many directions.

We apply our result to an inverse Robin transmission problem with finitely many measurements that is motivated by EIT-based corrosion detection techniques. Using a relation to monotonicity and localized potentials arguments, we will show that the discretized inverse Robin transmission problem always fulfills our criterion if enough measurements are being used. Thus, our result enables us to determine those boundary measurements from which an unknown coefficient can be uniquely and stably reconstructed with a given desired resolution by a globally convergent Newton iterationIn discussing weak unique continuation of our operator, a main feature of our argument relies on

a Carleman estimate for the associated fractional parabolic Caffarelli-Silvestre extension. Furthermore, we also discuss constructive single measurement results based on the approximation and unique continuation properties of the equation.

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THE INVERSE CONDUCTIVITY PROBLEM FOR COMPLEX CONDUCTIVITIES WITH REGULAR JUMPS

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Abstract

In this talk we will introduce some new ideas for the inverse conductivity problem in the case where the conductivity is complex and has a regular jump [1]. This material has never been considered in literature and the best result assumed Lipschitz complex conductivities.

For the study of this problem we model it as an interior transmission problem. We will introduce several new concepts that were required to treat this problem. One of them is the set of admissible points, which is essential for us since it permits the enlargement of the set of CGO waves. This will be the key point to obtain a reconstruction formula for the conductivity in this set of points. Even being apparently a rather weak result, it cannot possibly be obtained by any previous technique, at least that we know of. Therefore, these ideas represent a new step in this direction and due to being early results we will present some of the footwork necessary to proceed further.

Future ideas are to establish this approach in three dimensions by usage of quaternionic analysis.

References

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INTERIOR DECAY OF SOLUTIONS TO ELLIPTIC EQUATIONS

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Abstract

We investigate the decay in the interior of solutions to elliptic equations. We show that the decay depends on the properties of the corresponding boundary data, in particular the decay rate is related to the so-called frequency of the boundary datum. This result has interesting applications to the choice of optimal measurements for electrical impedance tomography.

This is a joint work with Michele Di Cristo.

GENERATIVE MODELS FOR ELECTRICAL IMPEDANCE TOMOGRAPHY

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Abstract

Electrical impedance tomography (EIT) is an imaging modality which consists in the determination of the conductivity distribution of a body from electrode measurements acquired on its boundary. The inverse problem is severely ill-posed, which results in very low resolution reconstructions and high instability with respect to noise and modelization errors. In recent years, several machine learning approaches have been able to mitigate the ill-posedness of EIT. Most of these results rely on training datasets with rather specific structure.

In this talk we will present a new data-driven approach based on generative models, which allows us to consider training datasets with higher variability. Taking inspiration from well-known architectures (e.g. U-Net), we construct and explicitly characterize a class of injective generative models defined on infinite-dimensional functions spaces. After an off-line training of the generative model, the proposed reconstruction method consists in an iterative scheme in the low-dimensional latent space. The main advantages are the faster iterations and the reduced ill-posedness, which is shown with new Lipschitz stability estimates.

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MINISYMPOSIUM
M13: Inverse Problems in Geomathematics and Seismology

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This minisymposium is concerned with mathematical inverse problems arising from geophysics and seismology. An important application that we want to address is the fault reconstruction problem in seismology: the geometry and the position of the fault are unknown, and the impulse on the fault producing measurable surface displacements is also unknown. This minisymposium will bring together analysts, geophysicists with direct experience in seismic modeling and data processing, and investigators who worked on related computational methods.

This minisymposium will spawn discussions on models as well as mathematical and computational tools– and together we will identify future research directions.

A STABILITY ESTIMATE FOR AN INVERSE ELECTROSEISMIC PROBLEM

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Abstract

In porous media, the motion of charges at the fluid/solid interfaces induces a coupling between seismic and electromagnetic waves. This phenomenon, known as the electro-kinetic effect is at the heart of electroseismic imaging, a technique used in oil prospection to image sedimentary layers of porous media. It combines the high sensitivity to material parameter contrast of electromagnetic waves and the ability to carry information with little distortion of seismic waves.

The system we study was proposed by Pride [1]. It couples the Biot equations, which govern the propagation of elastic waves through porous media, and the Maxwell equations in a bounded domain in $\mathbf{R}^3$. As the electo-kinetic coupling is weak, we only consider the EM to seismic wave conversion in which no coupling term appears in the equations for the electromagnetic fields (electroseismic model). The objective is to recover the electromagnetic parameters of the medium and the electrokinetic coupling constant from measurements of the electric fields and of the dispacements of the solid and the fluid in a neighborhood of the boundary.

For the electroseismic system, assuming that the physical parameters are sufficiently smooth, we prove a Carleman estimate, that induces the Hölder stability of the inverse problem [2].

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EARTHQUAKE INUCLEATION: DIRECT AND INVERSE PROBLEMS

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Abstract

By slow event, we mean important slip taking place on an intermediate time scale (i.e. minutes to months). This is much longer than seismic time scales (seconds) but much shorter than geological time scales (hundreds of years). Since slow slip events are aseismic (i.e., there is no associated seismic wave), their detection is very difficult. Two types of phenomena can be related to slow slip events: silent earthquakes and nucleation (or initiation) phases for (ordinary) earthquakes. The earthquake nucleation (or initiation) phase, which precedes dynamic rupture, was uncovered by detailed seismological observations and important physical properties of the nucleation phase (characteristic time, critical fault length, etc.) were obtained through simple mathematical properties of unstable evolution (see for instance [1]). Early detection of the nucleation phase from surface displacements has the potential to play a key role in short time prediction of large earthquakes.

Second, we focus on the material damage associated to slow slip events. The existence of this damagedzone is traditionally evidenced at depth by seismic tomography that indicates a lower velocity zone extending several kilometers in depth. A noise based passive monitoring allows to envision a long-term continuous monitoring of the seismic velocity. Examples of these changes are now well documented and it is possible to detect them continuously with an unprecedented precision reaching a few 10-5 for relative velocity changes and very short time resolution. An important issue is to explain how very small macroscale deformation, associated to the tectonic deformation, could induce observable velocity drop. The study of the wave velocity drop through a numerical experiment with cohesive discrete media show a large sensitivity due to strain field heterogeneity [3]. Moreover, wave velocities monitoring in these media can measure subtle processes such as earthquake initiation phases. Remarkably, our simple model indicates a large "susceptibility" as that observed in real data.

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COUPLED FLOW AND GEOMECHANICS IN FRACTURED POROUS MEDIUM

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Abstract

Numerous applications of subsurface engineering involve injection and extraction of fluids. Examples include geothermal energy extraction, nuclear waste storage, carbon sequestration, petroleum engineering applications, and energy storage. These anthropogenic activities involve a complex set of processes involving flow, thermal, chemical reactions, and mechanical effects all possibly coupled to each other. These complex sets of processes interact with the complex geology that involves ubiquitous fractures and faults. The network of fractures forms the primary conduit of flow and transport and furthermore, act as the most vulnerable regions for mechanical instability. The interaction of processes and the complex geometry of fractures brings computational and mathematical challenges in the simulation of these processes. The fractured medium is generally anisotropic, heterogeneous, and has substantially discontinuous material properties spanning several orders of magnitude.

Our objective is to study coupling of flow and geomechanics in a fractured porous medium setting. We present a mixed dimensional model for a fractured poro-elasic medium. The fracture is a lower dimensional surface embedded in a bulk poro-elastic matrix. The flow equation on the fracture is a Darcy type model that follows the cubic law for permeability. The bulk poro-elasticity is governed by fully dynamic Biot equations. The resulting model is a mixed dimensional type where the fracture flow on a surface is coupled to a bulk flow and geomechanicsmodel.

There are two directions in which our work contributes to. The first is in extending Biot equations to include fracture flow model and complex friction and contact mechanics. The second is in considering different time schemes for the Multiphysics modelling. We consider finer time steps for the flow and coarser time steps for the mechanics.

This is joint work with Tameem Almani (Aramco), Maarten de Hoop (Rice), Vivette Girault (Paris 6), and Mary F Wheeler (Austin), Ruichao Ye (Chevron).

DEEP LEARNING ARCHITECTURES FOR NONLINEAR OPERATOR FUNCTIONS AND INVERSE PROBLEMS FOR WAVE EQUATIONS

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Abstract

We consider a new type of deep neural network developed to solve nonlinear inverse problems. In particular, we consider inverse problems for a wave equation where one want to determine an unknown wave speed from the boundary measurements. In particular, we consider the model where the wave propagation is governed by the linear acoustic wave equation on an interval. A novel feature of the studied neural network is that the data itself form layers in the network. This corresponds to the fact that data for the inverse problem is a linear operator that maps the boundary source to the boundary value of the wave that is reflected from the unknown medium. Even though the wave equation modelling the waves is linear, the inverse problem of finding the coefficients of this equation is non-linear. Using the classical theory of inverse problems we design a neural network architecture to solve the inverse problem of finding the unknown wave speed. This makes it possible to rigorously analyze the properties of the neural network.

For inverse problems, the main theoretical questions concern uniqueness, range characterisation, stability and the regularisation strategies for the inverse problems. We will discuss the question when a solution algorithm generalises from the training data, that is, when the solution algorithm trained with a finite number of samples can solve the problem with new inputs that are not contained in the training data. This can viewed as a new question for classical inverse problems that takes its motivation from machine learning.

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REGULARIZATION FOR SEISMIC SOURCES INVERSION FROM INTERFEROMETRIC DATA

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Abstract

In this talk, we deal with the inverse source problem from interferometric measurements. Interferometric data are made of correlations of the wavefield recorded at neighboring positions and frequencies. In some configurations, these correlations data are very stable with respect to phase shift due to some wave-speed uncertainties inside the medium. Hence, trying to recover the source from this data is of large interest. Some well known works describe techniques of "interferometric migration", inspired by the classic back-propagation technique, to form an image of the source from this data. A refocalization phenomenon is observed and the built image can be much better that the classic back-propagation of the wavefield measurements.

The inverse problem from interferometric measurements is challenging as the system of equations to solve becomes a quadratic system. In the sense of least squares, the objective function is then a non convex 4-th order polynomial.

We will show that it is possible to solve this problem using an additional regularization term and how some error estimates can be proved. In various numerical applications, we will illustrate the stability of the reconstruction with respect to some important medium uncertainties leading to an important phase shift. This technique as a wide potential in other fields such as signal and image denoising problems or the phase retrieval problem.

STABILITY PROPERTIES FOR A CLASS OF INVERSE PROBLEMS WITH APPLICATIONS TO NEURAL NETWORK SOLUTIONS

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Abstract

We establish Lipschitz stability properties for a class of inverse problems. In that class, the associated direct problem is formulated by an integral operator $\mathcal{A}_m$ depending non-linearly on a parameter m and operating on a function u. In the inversion step both u and m are unknown but we are only interested in recovering m. We discuss examples of such inverse problems for the elasticity equation with applications to seismology and for the inverse scattering problem in electromagnetic theory. Assuming a few injectivity and regularity properties for $\mathcal{A}_m$, we prove that the inverse problem with a finite number of data points is solvable and that the solution is Lipschitz stable in the data.

We show a reconstruction example illustrating the use of neural networks

BAYESIAN INFERENCE OF SEISMIC EVENTS FROM LOCAL DEFORMATIONS USING THE WEAKLY-ENFORCED SLIP METHOD

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Abstract

Earthquakes cause lasting changes in static equilibrium, resulting in global deformation fields that can be observed. Consequently, deformation measurements such as those provided by satellite based InSAR monitoring can be used to infer an earthquake's faulting mechanism. This inverse problem requires a numerical model that is both accurate and fast, as typical inverse procedures require many forward evaluations. The Weakly-enforced Slip Method (WSM) [1] was developed to meet these needs, but it was not before applied in an inverse problem setting. In this contribution we show that the WSM is able to accurately recover slip distributions in a Bayesian-inference setting.

The WSM is a Galerkin method: given stiffness tensor C, slip vector b and fault plane normal vector ν, the WSM displacement field u, is that which for any test function v satisfies:

$$\int_\Omega \nabla u : C : \nabla v = \int_F b \cdot \{C : \nabla v\} \cdot \nu \qquad (1)$$

Crucially, the left hand side of this equation translates to the standard stiffness matrix for linear elastic computations. This means that expensive steps such as meshing, assembly, and factorization are independent of the fault geometry and can all be reused. The novelty of the WSM lies in the right hand side, which involves an integral over the fault plane alone, and as such is relatively cheap to evaluate. The result is a method that has the same up front costs as an ordinary elasticity computation but comparatively low costs per fault. Compared to conventional solution strategies, the main downside of the WSM is that the solution space cannot account for discontinuities of the displacement field. Instead, WSM solutions feature sharp gradients in the vicinity of the fault as approximations of a jump. While this implies a large error local to the fault, this error is shown [1] to decay rapidly with distance, and to converge optimally under mesh efinement.

To study the WSM in an inverse setting we use the following approach. First, we synthesize deformation data based on analytical solutions for a (2D and 3D) homogeneous half-space and add measurement noise. Then, we use Bayesian inference to form a posterior probability of the faulting mechanism, using the same forward model to establish the theoretical optimum. Finally, we repeat the inversion using the WSM forward model to identify the influence of the WSM's error components and develop potential mitigating practices. In a representative scenario, we found a finite element size of 1 km to be sufficiently fine to generate a posterior probability distribution that is similar to the theoretical optimum. Buried faults of moderate depth required no special intervention, as the surface is everywhere removed from the dislocation. Rupturing faults, on the other hand, require a masking zone of 10 km to avoid numerical disturbances that would otherwise be nduced by the local discretization error. Errors induced by boundary treatment were automatically mitigated by treating the displacement data as inherently relative.

Our results demonstrate that the WSM is a viable forward method for earthquake inversion problems. While our synthesized scenario is basic for reasons of validation, our results are expected to generalize to the wider gamut of scenarios that finite element methods are able to capture. This has the potential to bring modeling flexibility to a field that is often forced to impose model restrictions in a concession to computability.

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MINISYMPOSIUM
M14: Mathematical Methods in Tomography Across the Scales

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Tomographic imaging continues to attract a lot of interest from the inverse problems community because of the increasing need of new mathematical models describing better the experiments and of efficient and sophisticated computational algorithms handling big data. In Austria, five Universities and one institute are collaborating in this direction under the Special Research Project "Tomography across the scales" founded by the Austrian Science Fund (FWF).

In this mini-symposium we bring together researchers, members of the project and external collaborators, working on imaging problems from the nanoscale of single molecule imaging to microscale of multi-modal imaging. We cover topics such as adaptive optics, quantitative reconstructions, image processing, and integral transforms.

DATA-DRIVEN REGULARIZATION

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Abstract

In this talk I will speak about some recent results on the study of linear inverse problems under the premise that the forward operator is not at hand but given indirectly through some input-output training pairs. We show that regularisation by projection and variational regularisation can be formulated by using the training data only and without making use of the forward operator. We will provide some information regarding convergence and stability of the regularized solutions. Moreover, we show, analytically and numerically, that regularisation by projection is indeed capable of learning linear operators, such as the Radon transform.

This is a joint work with Yury Korolev (University of Cambridge) and Otmar Scherzer (University of Vienna and RICAM).

QUANTITATIVE SEISMIC IMAGING USING RECIPROCITY-BASED METHODS ENABLING ARBITRARY PROBING SOURCES

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Abstract

We study the inverse problem associated with the propagation of time-harmonic waves for seismic applications, where we work with partial reection data. The recovery of the subsurface Earth medium parameters is conducted using an iterative minimization procedure of a mis_t functional.

Following the measurements of two types of data (e.g., with dual-sensors devices), we can define a new misfit functional based upon the reciprocity-gap. The main feature of this misfit functional is to allow a separation between the observational and numerical sources. Namely, the position of the observational sources do not need to be known, and arbitrary probing sources can be used for the numerical simulations. In particular, it o_ers the possibility to create adapted computational acquisitions in order to reduce the numerical burden.

We shall present the method for both acoustic and elastic wave problems, and illustrate the robustness of our approach with respect to shot stacking with three-dimensional experiments.

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DATA-DRIVEN METHODS IN INVERSE PROBLEMS

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Abstract

We will focus on data-driven regularization by projection showing some numerical results and comparisons with different methods. We will also discuss in details "Seidman's non-convergence example on regularization". Additionally, we provide an application of a projection algorithm, utilized and applied in frames theory, as a data driven reconstruction procedure in inverse problems.

A FRAME DECOMPOSITION OF THE ATMOSPHERIC TOMOGRAPHY OPERATOR

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Abstract

We consider the problem of atmospheric tomography, as it appears for example in adaptive optics systems for extremely large telescopes. A singular-value-type decomposition of the underlying atmospheric tomography operator has been derived previously, for setups with only natural guide stars and square aperture shapes.

We extend these results by deriving a frame decomposition of the atmospheric tomography operator which allows both mixed natural and laser guide star setups, as well as arbitrary aperture shapes.

The significance of the derived results is discussed both from an analytical and a numerical perspective.

A PROBABILISTIC ORACLE INEQUALITY AND QUANTIFICATION OF UNCERTAINTY OF A MODIFIED DISCREPANCY PRINCIPLE FOR STATISTICAL INVERSE PROBLEMS

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Abstract

In this talk we consider spectral cut-off estimators to solve a statistical linear inverse problem under arbitrary white noise. The truncation level is determined with a recently introduced adaptive method based on the classical discrepancy principle. We provide probabilistic oracle inequalities together with quantification of uncertainty for general linear problems. Moreover, we compare the new method to existing ones, namely early stopping sequential discrepancy principle and the balancing principle, both theoretically and numerically.

References

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A FOURIER APPROACH TO THE INVERSE SOURCE PROBLEM IN AN ABSORBING AND SCATTERING MEDIUM WITH APPLICATIONS TO OPTICAL MOLECULAR IMAGING

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Abstract

We revisit the inverse source problem in a two dimensional absorbing and scattering medium and present a direct reconstruction method, which does not require iterative solvability of the forward problem, using measurements of the radiating flux at the boundary. The approach is based on the Cauchy problem for a Beltramilike equation for the sequence valued maps, and extends the original ideas of A. Bukhgeim from the non-scattering to the scattering media. Of novelty here, the medium has an anisotropic scattering property that is neither negligible nor large enough for the diffusion approximation to hold.

The numerical realization of the proposed reconstruction method is also presented, which is amenable for such scattering media. The feasibility of the proposed algorithm is demonstrated in several numerical experiments, including simulated scenarios for parameters meaningful in Optical Molecular Imaging.
This is joint work with Alexandru Tamasan and Hiroshi Fujiwara.

DISPLACEMENT FIELD ESTIMATION UTILIZING SPECKLE INFORMATION FOR PARAMETER RECOVERY IN QUANTITATIVE ELASTOGRAPHY

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Abstract

Diseases like cancer and arteriosclerosis often cause changes of tissue stiffness in the micrometer scale. Thus, it is possible to identify malignant formations inside tissues by means of quantitative elastography. As a medical imaging modality, elastography examines the tissue by deforming it mechanically to produce qualitative or desirably quantitative maps of the tissue. In this work, we consider two successive, closely related problems which arise as we attempt to quantitatively recover the unknown biomechanical parameters of a tissue. First, the inversion method we use for finding the parameters [1] relies on the knowledge of the internal displacement field which occurs in the tissue due to deformation. In the considered physical experiments, successive OCT/PAT scans of the object interior are available before and after compression which are typically used as the input for the displacement estimation. In our case, additional speckle information extracted from the OCT scans is used for enhancing the quality of the displacement reconstruction.

We present and analyse a framework for including the speckle information in the estimation procedure. Second, we are interested in restoring elastic material parameters from the obtained displacement field to demonstrate the practical usefulness of the proposed approach with speckle data.

In particular, we present numerical results for the displacement and parameter estimation based on both simulated and experimental data.

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QUANTITATIVE OCT RECONSTRUCTIONS FOR DISPERSIVE MEDIA

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Abstract

Optical coherence tomography(OCT) is a non-invasive imaging technique, which produces high-resolution images of the inner structure of biological tissues by measuring the intensity of the backscattered light from the sample. In this talk we discuss the inverse scattering problem of extracting quantitative information about the underlying object in form of reconstructing its optical properties from OCT measurements. We consider a linear dispersive medium with multi-layered structure, described by a frequency- and depth-dependent parameter, the electric susceptibility. The parameter identi_cation problem can then be formulated as a one-dimensional inverse problem. Furthermore, an iterative scheme, based on the solution of Helmholtz equation, which is the core of the reconstruction algorithm for the frequency dependent parameter will be also presented.

This is joint work with Peter Elbau and Leonidas Mindrinos.

MINISYMPOSIUM
M15: Inverse Source Problems with Applications to Planetary Sciences and Medical Imaging

Organizer:
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Inverse Poisson problems with source term in divergence form is a class of inverse problems occuring naturally in Electromagnetics, under quasi-static assumptions. They arise for instance in connection with Inverse Magnetization Problems, e.g. in Paleomagnetism or Geomagnetism, as well as in Electro-Encephalography or Magneto-Encephalography. In recent years, tools from harmonic analysis, in particular extremal problems in Hardy spaces of harmonic gradients, as well as notions of sparsity from geometric measure theory, have been applied to recovery problems, moment estimation, and source separation.

INVERSE PROBLEM OF SOURCE IDENTIFICATION IN ELECTROENCEPHALOGRAPHY (EEG)

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Abstract

We aim to solve the problem of source identification in EEG by modeling the sources as $\boldsymbol{R}^3$ -valued finite linear combinations of dipoles. To achieve this we formulate the problem over the Banach space of $\boldsymbol{R}^3$ -valued Radon measures, compactly supported in a ball. This problem is ill-posed hence we minimize a Tikhonov type problem with a "weighted" total variation regularization term, to achieve dipole recovery.

We discuss constructive issues, using an algorithm developed by Bredies and Pikkarainen [1], and suggest a criterion for an appropriate choice of the "weight" towards a good dipoles recovery.

References

[1] K. Bredies and H. Pikkarainen. Inverse problems in spaces of measures. *European Series in Applied and Industrial Mathematics* (ESAIM): Control, Optimization and Calculus of Variations, 19, 12 2010.

SOME INVERSE MAGNETIZATION PROBLEMS IN GEOSCIENCE

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Abstract

Potential Field Problems, in particular those concerning the Earth's gravity and magnetic field, have a long history in geophysics. Nowadays, corresponding data is available at a variety of scales, ranging from microscopy data over surface and airborne data to global satellite data. In this presentation, we want to provide an overview on some inverse problems arising with magnetic field data, of which further details are discussed in accompanying presentations.

Topics that will discussed are uniqueness issues for the recovery of magnetic sources in a volumetric as well as surface setup, separation of magnetic sources, and possibly connections to other geophysical quantities (like geothermal heat flow).

SOME MEASURE-THEORETIC ASPECTS OF PLANAR MAGNETIZATION RECONSTRUCTION

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Abstract

When modeling magnetizations by R^3-valued Borel measures, the kernel of the forward operator, mapping magnetizations supported on the plane {z = 0} to the restriction of the magnetic fields they induce to a sufficiently dense subset of an analytic surface, consist of divergence free measures whose 3rd component is zero (see [1] and [2]). Using this fact as a motivation, we will show how any divergence free R^2-valued Borel measure on R^2 can be decomposed into line integrals over Jordan curves.

This result has two important implications for the recovery of planar magnetizations. First, that any magnetization supported on sufficiently separated straight lines has minimal total variation, in the measure-theoretic sense, among all other planar magnetizations that induce the same field. Second, that the group LASSO regularization problem in this context, which amounts to total variation regularization, has a unique solution for any value of the regularization parameter.

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RELATING HARDY COMPONENTS OF COMPACTLY SUPPORTED MAGNETIZATIONS ON SPHERE

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Abstract

The null space of the magnetic potential with sources supported on a spherical membrane can be characterized by the Hardy-Hodge decomposition. Moreover, if the underlying magnetization is spatially localized in a subdomain of the sphere, there exists a unique map between its inner and outer Hardy components. Using layer potentials, we will dicuss the contruction of this map and its relation to Cauchy problems, characterize its domain, and prove some further properties of interest for related bounded extremal problems.

This is joint work with Christian Gerhards and Alexander Kegeles.

A "SIMPLE" APPROACH TO THE MAGNETO-STATIC INVERSE PROBLEM

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Abstract

In geomagnetism, we often encounter the following inverse problem: A magnetic source creates a surrounding magnetic potential; we measure that potential and seek the source magnetization.

In general, this problem is ill-posed. In particular, it has no unique solution on the space of square-integrable vector fields.

In this talk, we will characterize the non-uniqueness of this problem. We will show that a part of the magnetization can be uniquely recovered, but this part has only limited (geo)physical use. Nevertheless, despite the ill-posed nature of the problem, we also show that the inversion is unique for a dense subspace of square-integrable vector fields. This subspace rests upon mild physical assumptions and has, quite literally, a simple mathematical description.

A LAYER POTENTIAL APPROACH TO FUNCTIONAL AND CLINICAL BRAIN IMAGING

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Abstract

We consider the inverse source recovery problem from stereo-ElectroEncephaloGraphy (sEEG), ElectroEncephaloGraphy (EEG) and MagnetoEncephaloGraphy (MEG) point-wise data. We regard this as an inverse source recovery problem for $[X]^3$ -valued vector-fields, where X is a Banach space of source terms supported on the grey/white matter interface within the brain, which together with the skull and the scalp form a non-homogeneous layered conductor.

We assume that the quasi-static approximation of Maxwell's equation holds for the electro-magnetic fields considered. The electric data is measured point-wise inside (sEEG data) and outside the conductor (EEG data) while the magnetic data (MEG data) is measured only point-wise outside the conductor. This ill-posed problem is solved via Tikhonov regularization on triangulations of the interfaces and a piecewise linear model for the current on the triangles. Both in the continuous and discrete formulation the electric potential is expressed as a linear combination of the Newton potential of the (distributional) divergence of the source and double layer potentials while the magnetic flux density in the continuous case is a vector-surface integral whose discrete formulation features single layer potentials. A main feature of our approach is that these contributions of the single and double layer potentials can be computed exactly, [1]. Regularity conditions for the electric potential in the inverse source recovery problem allow the associated Cauchy transmission problem to be inadvertently solved as well. For the latter, we only propagate the electric potential while the normal derivatives (current flux) at the interfaces of discontinuity of the electric conductivities are computed directly from the resulting solution. This reduces the computational complexity of the problem. Because of the connection between the magnetic flux density and the electrical potential in conductors such as the one we explore, a coupling of the sEEG, EEG and MEG data for solving the respective inverse source recovery problems simultaneously is direct, [2]. We treat these problems in a unified approach that uses single and/or double layer potentials. To solve the associated Tikhonov problems we use an alternating minimisation procedure that alternates between refining the source localisation and the cortical mapping, [3].

Numerical experiments were performed using meshes of realistic head geometries with synthetic data and the results will be discussed..

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MINISYMPOSIUM

M16: Recent Advances in Inverse Problems and Distributed Parameter Systems

Minisymposium in honor of Professor H. T. Banks' long career in applied mathematics and his leadership in inverse problems

Organizers:
John A. Burns, Interdisciplinary Center for Applied Mathematics, Virginia Tech., USA, jaburns@vt.edu
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Since the middle of the 1970's, Professor H. Thomas Banks, has been one of the world's most distinguished and active researchers in the area of parameter identification and inverse problems for partial differential equations. Mathematical topics to which H.T. Banks has made contributions (many motivated by applied problems) include: control of delay and partial differential equations, approximation and computational methods, identification and inverse problem methodology for linear and nonlinear distributed parameter systems, nonlinear damping and hysteresis modeling, homogenization, and the mathematics of smart material structures. Service to the profession has always been an important obligation for H.T. Banks. He has received generous recognition for his service and research efforts with students, postdocs and colleagues. These honors include Professor Honoraire, Universite de Compaigne, 1977; appointed University Professor and Drexel Professor of Mathematics, NCSU, 1992; elected Fellow, IEEE; Fellow, Institute of Physics,; Fellow, SIAM; Fellow, AAAS; IEEE-CSS Control Systems Technology Award, 1996; Distinguished Alumni Award, Purdue Univ., 1998; named Alumni Distinguished Graduate Professor, NCSU, 2000. He has served on more that 20 editorial boards over the past 20 years.

In this minisymposium we will recognize his accomplishments with presentations by a variety of scientists on topics which are connected to and influenced by the pioneering work of Professor Banks.

FINITE DIFFERENCE SCHEMES FOR A STRUCTURED COAGULATION FRAGMENTAITON MODEL IN THE SPECE OF MEASURES

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Abstract
We formulate a structured coagulation-fragmentation model on the space of Radon measures equipped with the bounded Lipschitz norm. This formulation unifies the study of discrete and continuous coagulation-fragmentation models under one framework. We prove that the model is well-posed and show that it can reduce down to the classical discrete and continuous coagulation-fragmentation models. To understand the interplay between the physical processes of coagulation and fragmentation and the biological processes of growth, reproduction, and death, we establish a regularity result for the solutions and use it to show that stationary solutions are absolutely continuous under some conditions on model parameters. We then develop and compare multiple finite difference schemes. In particular, we develop a fully explicit scheme, a semi-implicit scheme, and an explicit scheme based on the mass conservation law governed by the model. We prove convergence for each scheme and test the performance of the schemes against multiple examples.

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[3] A.S. Ackleh, R. Lyons, and N. Saintier. Finite difference schemes for structured coagulation-fragmentation equation in the space of radon measures. submitted.

POPULATION DYNAMICS IN APPLIED ECOLOGY: MODELS & EXPERIMENTS

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Abstract

Understanding how and why populations fluctuate is a central concern in applied ecology. Mathematical models, especially those parameterized with field- and laboratory-derived data, are a powerful tool for understanding population dynamics and the forces governing population regulation. I describe several examples taken from nearly two decades of collaborative research with the H.T. Banks lab in which field/lab data from ecological experiments were incorporated into population models in order to estimate parameters and derive insights into drivers of population conservation or control.

Case studies taken from ecological field studies include research around issues of forest bird conservation as well as ecosystem services such as pest control and pollination in agricultural systems.

Finally, I address the overall utility of using inverse-problem approaches for parameter estimation along with delay differential equations to better understand population dynamics in applied ecology.

NUMERICAL METHODS FOR CONTROL, OPTIMIZATION, DESIGN AND ESTIMATION OF INFINITE DIMENSIONAL SYSTEMS

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Abstract

This study is motivated by sensor location problems for infinite dimensional linear distributed parameter systems (DPS) defined by partial and delay differential equations (PDEs). The term "sensor location" usually refers to a spatial location. The problem of "optimal" sensor (and actuator) location plays an important role in controller design, system identification and state estimation and, depending on the application, can be formulated in many ways. Any optimal location problem implies that there is a cost function to be minimized and the choice of this cost function places specific requirements on the computational methods needed to solve the problem. In addition, choices such as "optimizing observability" can lead to ill-posed problems and result in non-convergence of suboptimal placements. On the other hand, minimum error variance methods (Kalman filtering) can be shown be well-posed. Efficient computational methods for solving the corresponding Riccati partial differential equation have been developed and are readily available.

In this presentation we focus on state estimators based on the Luenberger observer and suggest some natural cost functions for sensor location. The framework is fairly general and applies to both finite and infinite dimensional systems.

Examples are given to illustrate the ideas.

WHY ARE THERE SO MANY STRUCTURAL HEALTH MONITORING ALGORITHMS?

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Abstract

H.T Banks taught me the importance of existence and uniqueness in trying to solve damage detection problems which later become the area of structural health monitoring. It often happens in science, engineering and mathematics that different discipline specific researchers ignore problems and techniques in those disciplines not directly related to their own.

The purpose of this review recognize the important similarities between various disciplines and that these similarities can be exploited to provide new results. Specifically some forms of structural health monitoring are very much like several other disciplines. In order to limit the scope of this presentation it is limited to structures describing linear structural systems that can be well defined in second order form resulting from the direct application of Newton's law. Four problems are considered that are mathematically identical. They are: inverse eigenvalue problems from mathematics, eigenstructure assignment from controls, model updating (also called model correction) from structural dynamics and structural health monitoring, largely from the fields of structural dynamics and signal processing.

INVERSE PROBLEM FOR ELECTROMAGNETIC PROPAGATION IN HUMAN MUSCLE TISSUES USING HAMILTON MONTE CARLO METHOD

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Abstract

Recently, electromagnetic imaging is of considerable interest in the signal detection and the identification related to the development of new ceramic materials, the early detection of anomalies in polymer, metabolic functioning of human bodies, etc [1]. We consider an inverse problem arising in human muscle tissues based on frequency dependent dielectric media using Debye formulation. Let $c = (\mu_0 \varepsilon_0)^{-0.5}$ be the speed of light and let $\{k_i = \omega_i /(2\pi c)\}$ be the interrogation wavenumber with respect to the specified angular frequency ω_i. Setting as $\eta_0 = \mu_0 / \varepsilon_0$, the complex permittivity is represented by

$$\hat{\varepsilon}_r(k_i) = \varepsilon_\infty + \frac{i\eta_0\sigma_0}{2\pi k_i} + \sum_{m=1}^{M} \frac{\Delta\varepsilon_m}{1 + i(2\pi c\tau_m)k_i}, \qquad (1)$$

where

$$\Delta\varepsilon_m = (\varepsilon_s - \varepsilon_\infty) g_m. \qquad (2)$$

The reflective coefficient r at the plane interface between air and a dielectric medium is given by

$$r(k_i) = \frac{1 - \sqrt{\hat{\varepsilon}_r(k_i)}}{1 + \sqrt{\hat{\varepsilon}_r(k_i)}_i}, \qquad (3)$$

Thus, measurements by time domain reflectometry method (TDR) provide the observable reflectance given by

$$R(k_i) = r(k_i)\ \overline{r(k_i)}, \text{ for } i = 0, 1, \ldots, N-1 \qquad (4)$$

Our inverse problem treated here is to identify the unknown parameter vector

$$q^M = \left\{k_0, \varepsilon_\infty, \varepsilon_s, \sigma_\varepsilon, \{\tau_k, \Delta\varepsilon_k\}_{k=1}^{K}\right\} \in R^M \text{ q}_M =$$

in (1)-(3) based on the model output (4).

An estimation algorithm can be performed by sampling procedures for the posteriori distribution from which sample paths can be drawn using Markov chains. In this paper, we demonstrate an estimation scheme using Hamiltonian Monte Carlo method (HMC) that is a Metropolis method making use of gradient information to reduce random walk behavior [2].

References

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2. R.M. Neal, MCMC using Hamiltonian Dynamics, *Handbook of Markov Chain Monte Carlo*, Eds. S. Brooks, A. Gelman, G. Jones, and X. Meng, Chapman & Hall, CRC Press, London, 2011.

MINISYMPOSIUM
M17: Parameter Identification Problems for PDEs: Theoretical and Computational Aspect

Organizer:
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Many phenomena in real-life are modeled by partial differential equations. Roughly they illustrate the relationships between result states, processes concerning derivatives and related parameters. Nevertheless, it might happen that some models are imprecise in the practical setting: parameters such as coefficients, source terms, boundary conditions may be subject to uncertainty which must be identified from measurement data of the states.

The minisymposium is a forum for scientists to present recent results on the uniqueness, stability and convergence of regularization methods for the parameter identification problem as well as on reconstruction algorithms and experimental implementations.

RECONSTRUCTION OF LAMÉ PARAMETERS IN LINEAR ELASTICITY

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Abstract

The main motivation of this problem is the non-destructive testing of elastic structures for material inclusions. We consider the inverse problem of recovering an isotropic elastic tensor from the Neumann-to-Dirichlet map.

We show that the shape of a region where the elastic tensor differs from a known background elastic tensor, can be detected by a simple monotonicity relation. In addition, we apply a linearization method in order to improve the qualitiy of the shape reconstruction and provide an insight into the numerics.

MODEL AND SOURCE IDENTIFICATION PROBLEMS FOR A SYSTEM OF ADVECTION-REACTION EQUATIONS AND APPLICATIONS IN WATER QUALITY

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Abstract

We discuss coefficient and source identification problems for a system of one-dimensional advection-reaction equations. The considered equations describe the transportation of pollutants in rivers or streams. We will discuss the stability using Carleman estimates and numerical methods for solving these inverse problems. We will demonstrate the performance of the proposed algorithms using simulated data generated using a realistic scenario.

This is a joint work with Dinh Nho Hao, Nguyen Van Duc, and Nguyen Van Thang.

The research of this talk is supported by Vingroup Innovative Foundation under grant number VINIF.2020.DA16.

ELECTRICAL IMPEDANCE TOMOGRAPHY WITH PARTIAL CAUCHY DATA

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Abstract

In this presentation we would like to present the problem of identifying the conductivity and/or the reaction coefficient in elliptic PDEs from several sets of Cauchy data on an accessible part of the boundary. The variational method of the finite element discretization combining with the regularization technique is applied to tackle the ill-posed identification problem.

The stability of the proposed approach and the convergence of the finite element regularization approximations to the sought coefficients are discussed, which confirm that the coefficients distributed inside the physical domain can be reconstructed from a finite number of partial Cauchy data.

A REDUCED BASIS ENSEMBLE KALMAN METHOD FOR INVERSE PROBLEMS

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Abstract

In the process of reproducing the state dynamics of distributed-parameter systems, data from physical measurements can be incorporated into the mathematical model to reduce the parameter uncertainty and, consequently, improve the state prediction. This process, referred to as Data Assimilation, must deal with the data and model misfit arising from experimental noise as well as model inaccuracies and uncertainties. In our study, we focus on the ensemble Kalman method (EnKM) [1], an iterative Monte Carlo method indicated for the a posteriori analysis of time series. The method is gradient free and, just like the ensemble Kalman filter, relies on a sample of parameters or particles ensemble to identify the state that better reproduces the physical observations, while preserving the physics of the system as described by the model.

In this context, for physical problems described by non-linear parabolic partial differential equations, we employ reduced order modeling (ROM) techniques [2] to generate surrogate models of different accuracy. These are combined with the EnKM to study its behavior in the presence of increasing levels of experimental noise and for model errors of different magnitude. We also experimentally investigate the role of the ensemble size on the reconstruction error and extend the method by implementing a poll-voting feature for improved parameter estimation. We emphasize that such experiments, involving unknown distributed parameters in two or more spatial dimensions, are potentially very expensive and are made possible by the remarkable efficiency provided by the surrogate models.

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MINISYMPOSIUM
M18: Coefficient Identification Problems

Organizer:
Daniel Lesnic, University of Leeds, UK, D.Lesnic@leeds.ac.uk

Coefficient identification problems occur naturally in almost any mathematical modelling of physical phenomena. They are ill-posed and in general nonlinear. Various aspects concerning the existence and uniqueness of solution are delicate to answer. Furthermore, the stability issue and the numerical reconstruction methods are of interest.

We want to bring together experts and young researches working in this field to discuss about new results in the analysis and numerics of coefficient identification problems.

AN INVERSE PROBLEM FOR THE STURM-LIOUVILLE EQUATION WITH ANALYTICAL DEPENDENCE ON THE EIGENPARAMETER IN A BOUNDARY CONDITION

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Abstract

The talk concerns the following eigenvalue problem:

$$-y''(x)+q(x)y(x)=\lambda y(x), \quad x\in(0,\pi), \tag{1}$$

$$y(0)=0, \quad f_1(\lambda)y'(\pi)+f_2(\lambda)y(\pi)=0. \tag{2}$$

Here (1) is the standard Shurm-Liouville equation with the complex-valued potential q from $L_2(0,\pi)$, but the boundary conditions (2) contain arbitrary functions f_1 and f_2 entire by the spectral parameter. We consider the inverse spectral problem, which consists in recovering the potential q from a part of the spectrum $\{\lambda_n\}_{n\geq 1}$, satisfying some conditions. The functions f_1 and f_2 are supposed to be known a priori.

Such inverse problem statement generalizes various inverse spectral problems applied in science and engineering:

- The Hochstadt-Lieberman problem [1];
- The inverse transmission eigenvalue problem [2];
- Partial inverse problems for Sturm-Liouville operators with discontinuities [3];
- Partial inverse problems for quantum graphs [4].

We will discuss the following issues of our inverse problem:

- Uniqueness;
- Constructive algorithm for solution;
- Sufficient conditions of solvability;
- Local solvability and stability.

Our approach is based on the methods of [5]. We reduce the considered inverse problem to the classical Sturm-Liouville inverse problem by two spectra, by using a special Riesz basis of vector-functions.

The work is supported by Grant 20-31-70005 of the Russian Foundation for Basic Research.

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INVERSE PROBLEMS TO RECONSTRUCT SPACE- AND TIME-DEPENDENT SOURCES IN EVOLUTION EQUATIONS CONTAINING FRACTIONAL LAPLACE OPERATORS

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Abstract

In the paper [1] we considered an inverse source problem for the equation

$$D_0^{\beta} u(x,t) = \Delta u(x,t) + F(x,t), \quad x \in \Omega \subset R^n, \quad t \in (0,T),$$

where D_0^{β} is Caputo or Riemann-Liouville fractional time derivative of the order $\beta \in R^+ \setminus N$ with lower terminal 0 and Δ the Laplacian. We assumed that $t \in (0,T)$ and supp $F \subset \Omega \times (0,t_0)$ and posed a problem to determine $u\big|_{\Omega\times(0,t_0)}$ and $F\big|_{\Omega\times(0,t_0)}$ on the basis of given $u\big|_{\Omega\times(t_0,T)}$. We proved uniqueness of a solution of this problem and deduced solution formulas. The non-locality of the time derivative D_0^{β} was an important prerequisite for achieving these results.

In this talk we consider an analogous problem in space directions. We follow the equation

$$D^{\beta} u(x,t) = -(-\Delta)^{\alpha} u(x,t) + F(x,t), \quad x \in R^n, \quad t \in (0,T)$$

where D^{β} is a time derivative of integer or non-integer order, $-(-\Delta)^{\alpha}$ is the fractional Laplacian of the order $\alpha \in (0,1)$ given by

$$(-\Delta)^{\alpha} v(x) + \Phi^{-1} |\xi|^{2\alpha} \Phi v(\xi),$$

and Φ is the Fourier transform, i.e.

$$\Phi v(\xi) = \int_{R^n} \exp(-i\xi \cdot x) v(x) dx$$

We assume that ω is an open subset of R^n and supp $F \subset (R^n \setminus \omega) \times (0,T)$ and formulate the following inverse problem: given $u\big|_{\omega\times(0,T)}$ find $u\big|_{(R^n-\omega)\times(0,T)}$ and $F\big|_{(R^n-\omega)\times(0,T)}$.

Reference

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RECOVERY OF A SPACE-DEPENDENT RATE OF REACTION IN A THERMAL-WAVE MODEL OF BIO-HEAT TRANSFER

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Abstract

This work investigates the numerical retrieval of the space-dependent blood perfusion coefficient (rate of reaction) in a newly derived dimensionless thermal-wave model of bio-heat transfer. The additional measurement for such reconstruction is the upper-base temperature or a pair of partial Cauchy data. The considered inverse problem is reformulated as a non-linear least-squares minimization problem. This minimization problem is solved by a minimization procedure based on the MATLAB routine lsqnonlin in conjunction with a direct solver of unconditionally stable finite difference method.

Several numerical tests for verifying the convergence and stability of the proposed algorithm are illustrated and discussed.

IDENTIFICATION OF HETEROGENEOUS MATERIAL COEFFICIENTS USING ULTRASONIC ARRAYS

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Abstract

Ultrasonic phased arrays are used for non-destructive imaging of safety critical structures such as those found in nuclear plants. Flaw imaging methods normally assume that the host material is a homogeneous solid and this can lead to reduced flaw detection and characterisation when the host material has heterogeneous material coefficients. Knowledge of the spatial variation in the material's internal elastic properties can therefore be used to improve flaw imaging. This paper will discuss techniques to reconstruct the spatially varying coefficients (the local orientation of polycrystalline materials) from ultrasonic phased array data. A Voronoi tessellation is used to parameterise the material's structure, where the coefficient of interest is constant in each cell. In the first instance, the reversible-jump Markov Chain Monte Carlo (rj-MCMC) method is used to sample the posterior distribution of this piecewise constant heterogeneous material.

In the second instance we solve this non-smooth, non-convex optimisation problem using a multi-start nonlinear least squares method on a square grid. Good reconstructions are achieved but the method is shown to be sensitive to the addition of noise. We prove that the orientations can be determined uniquely given enough boundary measurements and provide a numerical method that is relatively stable with respect to the addition of noise. The reconstructed material map is then used with an adapted flaw imaging algorithm and empirical results suggest that this produces more reliable reconstruction of defects.

AN ITERATION METHOD FOR SOLVING THE INVERSE PROBLEM OF FREEZING SOIL

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Abstract

At conductive heat transfer, the heat flow q is proportional to the temperature gradient $q = -k\nabla T$, where k – heat conductivity coefficient, T – temperature [1]. During the freezing process of soil, the following three zones are formed: thawed, phased and frozen zones. The fundamental law of conductive mechanism for heat transfer is valid for phased zone transitions. This leads to the conclusion that the heat conductivity coefficient $k(T)$ will change along with temperature. Assume that $k(T)$ – heat conductivity coefficients of thawed, phased and frozen soil; W_s – quantity of firmly bound water in soil; W – quantity of unfrozen water; θ_p – isotherm of phased zone; θ – isotherm of frozen zone. Moreover, $h_b(t)$ – boundary between thawed and phased zones, and $h(t)$ – boundary between phased and frozen zones. $c(T)$ – coefficient of volumetric heat capacity. We believe that at moving boundaries $h_b(t)$ and $h(t)$ conditions of temperature continuity are satisfied [2]: $T(h_b(t)+0,t) = T(h_b(t)-0,t) = \theta_p$, $T(h(t)+0,t) = T(h(t)-0,t) = \theta$, where $t > 0$ – time. By considering the features of physical process of moisture freezing in soil, we accept boundary conditions of 4th kind $[k\,\partial\theta/\partial z]_{(h_b(t))} = p(dh_b(t))/dt$, $[k\,\partial\theta/\partial z]_{(h(t))} = 0$. We also use the measured value of soil temperature at the Earth' surface. It is required to find such value of parameter k(T) that gives a minimum to the functional J(k).

In this presentation, an iterative method for calculating the coefficient $k(T)$ is developed. Moreover, based on experimental research done by scientists, a nonlinear model of temperature transfer in soil is built;

An iterative formula for calculating heat conductivity coefficient $k(T)$ is developed;

Limitation of approximate value of thermal conductivity coefficient $k(T)$ is proved, and the monotony of minimized functional $J(T)$ is verified;

Nonlinear difference problem is solved by the Newton method and method of choosing the initial approximation of the iterative process is indicated;

Quadratic convergence of the Newton method is proved;

Software program was written, and numerical calculations were conducted;

Analysis of convergence of the Newton method and assessment of the accuracy of the developed method were processed.

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UNIQUENESS FOR AN INVERSE SOURCE PROBLEM OF DETERMINING A SPACE DEPENDENT SOURCE IN A NON-AUTONOMOUS PARABOLIC EQUATION

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Abstract

We consider a bounded domain $\Omega \subset R^N$, $N \geq 1$ with su_ciently smooth boundary Γ. We are interested in the uniqueness of determining of an unknown couple $(u(x,t),\ f(x))$ obeying the following linear non-autonomous parabolic problem of second order

$$\begin{cases} u(x,t) + A(t)u(x,t) = h(t)f(x) + F(x,t), & in\ \ \Omega \times (0,T), \\ u(x,t) = g(x,t), & on\ \ \Gamma \times (0,T), \\ u(x,0) = u_0(x), & in\ \ \Omega, \end{cases} \tag{1}$$

where

$$u(x,T) = u_T(x). \tag{2}$$

Most of the known results deal with recovery for autonomous operator A, e.g. [1]. Isakov [2, Theorem 2.1] presented uniqueness result for $A(t)$ assuming some monotonic in time behavior of coeffcients at $A(t)$ and $0 \leq h(t)$, $0 \leq \partial_t h$. Slodicka and Johansson [3] also needed monotone in time assumption for $A(t)$ and

$$0 \leq h,\ \alpha'(t) \leq 0, \text{ where } \alpha(t) := \frac{h'(t)}{h(t)}.$$

The aim of this talk is to present some uniqueness results for the ISP just for $0 < h$ in the non-autonomous case $A(t)$.

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MINISYMPOSIUM

M19: Modern Challenges in Imaging, Tomography, and Radon Transforms

Minisymposium dedicated to the 70th anniversary of an outstanding expert in inverse problems and imaging sciences, Robinson Professor of Mathematics Tufts University, Todd Quinto

Organizers:
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Ming Jiang, Peking University, China, ming-jiang@pku.edu.cn (ask to remove, will not come)
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Imaging, Tomography and Radon Transforms are one of the fastest developing areas of inverse problems, and they have a wide range of applications. Tomography became generally known in the seventies when medical diagnosis was revolutionized by X-ray computed tomography. The mathematical model is an integral transform and Fourier integral operator--the Radon transform. Other imaging technologies lead to more complicated integral transforms as well as linear and nonlinear problems. Classical problems, such as limited-data tomography, are still challenging. Cone-Beam tomography, ultrasound tomography, multi-modal imaging, Compton and Bragg tomography, and time-dependent problems are other frontiers of research. All these technologies have in common that the mathematical models are ill-posed inverse problems.

The minisymposium celebrates one of the pioneers of the mathematics of imaging and tomography, Professor Eric Todd Quinto, on the occasion of his 70th birthday. It will bring together well-established scientists and young researchers to introduce new methods in the field, including the methods pioneered by Professor Quinto.

AN ADAPTIVE NONPARAMETRIC ESTIMATOR IN AN INVERSE PROBLEM FOR EXPONENTIAL RADON TRANSFORM

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Abstract

In this work, we propose a locally adaptive strategy for estimating a function from its Exponential Radon Transform (ERT) data, without prior knowledge of the smoothness of functions that are to be estimated. We build a non-parametric kernel type estimator and show that for a class of functions comprising a wide Sobolev regularity scale, our proposed strategy follows the minimax optimal rate up to a log n factor. We also show that there does not exist an optimal adaptive estimator on the Sobolev scale when the pointwise risk is used and in fact the rate achieved by the proposed estimator is the adaptive rate of convergence.

This is joint work with Dr. Sakshi Arya, Department of Statistics, Pennsylvania State University.

DOMAINS WITH ALGEBRAIC X-RAY TRANSFORM

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Abstract

Koldobsky, Merkurjev and Yaskin proved that given a convex body K in R^n, n is odd, with smooth boundary, such that the volume of the intersection of K with a hyperplane L (the sectional volume function) depends polynomially on the distance of L to the origin, then the boundary of K is an ellipsoid. In even dimension, the sectional volume functions are never polynomials, nevertheless in the case of ellipsoids their squares are. We conjecture that the latter property fully characterizes ellipsoids and, disregarding the parity of the dimension, ellipsoids are the only convex bodies with smooth boundaries whose sectional volume functions are roots (of some power) of polynomials and confirm this conjecture for planar domains, bounded by algebraic curves. A multidimensional version in terms of chords lengths, i.e., of the X-ray transform of the characteristic function, is given.

The result is motivated by Arnold's conjecture on characterization of algebraically integrable bodies.

MICROLOCAL ANALYSIS OF THE CROSSWELL AND THE WALKAWAY SEISMIC DATA

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Abstract

We study the microlocal properties of the forward operator, F (a Fourier integral operator), which maps the image to the data, in two special cases of the the borehole seismic data: crosswell and the walkaway geometry. In general, in the borehole seismic acquisition data, the receivers are located on a vertical borehole and the sources are in another borehole or on the surface. The relation between the singularities of the image and the singularities of the data is given by the canonical relation, C, of the forward operator, F. To understand the operator F, we consider the projections to the left and to the right of the canonical relation, C. We are also interested in the artifacts which appear by applying the backprojection operator, F*, to the forward operator (to find the image).

In the crosswell geometry, the sources are located in another vertical borehole, and in the walkaway geometry, the sources are on a line on the surface, passing over the borehole top. In both cases, we show that the canonical relation is singular at the union of two smooth hypersurfaces which intersect transversally. The singularities of the right and the left projections of the canonical relation, are folds and blowdowns. Also, in these two cases, we conclude that the artifacts are very strong.

This is joint work with A. Greenleaf, R. Gaburro and C. Nolan.

MICROLOCAL ANALYSIS IN TOMOGRAPHIC RECONSTRUCTIONS

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Abstract

During recent years, microloacal analysis has established itself as one of the standard tools for analyzing tomographic reconstructions. It has been shown that microlocal anaylsis is particularly useful for analyzing and understanding reconstrctions from incomplete data. In particular, using microlocal analysis one can gain valuable insights that allow for to a better interpreation of reconstruted images and that can be used to design dedicated algorithms for incomplete data problems. Specifically, microlocal analysis allows to characterize image features that can be realiably reconstructed from available tomographic data, and to explain why singular artifacts are generated in several classical imaging scenarios. Also, it provide strategies that allow to reduce or even avoid the generation of artifacts.

In this talk I will explain the basic concepts of microlocal analysis and review a framework for analyzing reconstructions in incomplete data tomography. In particular, I will discuss fundamental results that characterize visible singularities and singular artifacts. I will also outline how those results have contributed to the practical development of algorithm in different applications of tomography.

This talk is dedicated to Eric Todd Quinto in honor of his 70th birthday, who has made major contributions to both fields, mathematics of tomographic reconstruction and microlocal analysis.

RESOLUTION OF 2D RECONSTRUCTION OF FUNCTIONS WITH NONSMOOTH EDGES FROM DISCRETE RADON TRANSFORM DATA

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Abstract

Let f be an unknown function in $\mathbb{R}^2$, and f_ϵ be its reconstruction from discrete Radon transform data, where ϵ is the data sampling rate. We study the resolution of reconstruction when f has a jump discontinuity along a nonsmooth curve $\mathcal{S}_\epsilon$. The assumptions are that (a) $\mathcal{S}_\epsilon$ is an $O(\epsilon)$-size perturbation of a smooth curve $\mathcal{S}$, and (b) $\mathcal{S}_\epsilon$ is Holder continuous with some exponent $\gamma \in (0,1]$. We compute the Discrete Transition Behavior (or, DTB) defined as the limit $\mathrm{DTB}(\check{x}) := \lim_{\epsilon\to 0} f_\epsilon(x_0 + \epsilon\check{x})$, where $x_0$is generic. We illustrate the DTB by two sets of numerical experiments. In the first set, the perturbation is a smooth, rapidly oscillating sinusoid, and in the second - a fractal curve. The experiments reveal that the match between the DTB and reconstruction is worse as $\mathcal{S}_\epsilon$ gets more rough. This is in agreement with the proof of the DTB, which suggests that the rate of convergence to the limit is $O(\epsilon^{\gamma/2})$. We then propose a new DTB, which exhibits an excellent agreement with reconstructions. Investigation of this phenomenon requires computing the rate of convergence for the new DTB. This, in turn, requires completely new approaches. We obtain a partial result along these lines and formulate a conjecture that the rate of convergence of the new DTB is $O(\epsilon^{1/2}\ln(1/\epsilon))$.

REGULARIZATION BY ARCHITECTURE: LEARNING WITH FEW DATA AND APPLICATIONS TO CT

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Abstract

We start with a basic introduction on deep learning approaches to inverse problems. We then focus on the learned ISTA concept and describe it as a method for learning a data dependent optimized Tikhonov functional. The main part of the talk is on learning with few data. In particular we investigate deep prior networks for solving inverse problems. Using the LISTA architecture in a deep prior network allows to proof equivalences to classical regularization schemes. On the experimental side we focus on low dose CT reconstructions.

We present a standardized data set and perform a numerical comparison of different deep learning concepts. The comparison is in terms of accuracy but also in terms of the amount of test data needed for training. We close the talk with an overview of our plan for future research (DL for parametric PDEs, magnetic particle imaging).

This is the joint work with Johannes Leuschner, Maximilian Schmidt, Sören Dittmer, Daniel Otero Baguer, Clemens Arndt.

REGULARIZED RECYCLING METHODS FOR LINEAR INVERSE PROBLEMS WITH APPLICATIONS TO ADAPTIVE OPTICS

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Abstract

Subspace recycling techniques have been used quite successfully for the acceleration of iterative methods for solving large-scale linear systems. These methods often work by augmenting a solution subspace generated iteratively by a known algorithm with a fixed subspace of vectors which are ``useful' for solving the problem. Often, this has the effect of inducing a projected version of the original linear system to which the known iterative method is then applied, and this projection can act as a deflation preconditioner, accelerating convergence.

In this talk we consider subspace augmentation-type iterative schemes applied to a linear ill-posed problems in a continuous Hilbert space setting, based on a recently developed framework describing these methods. We show that under suitable assumptions, a recycling method satisfies the formal definition of a regularization, as long as the underlying scheme is itself a regularization. We then develop an augmented subspace version of the gradient descent method and demonstrate its effectiveness on problems arising in Adaptive Optics for the resolution of large sky images by ground-based extremely large telescopes.

QUANTITATIVE INVERSE PROBLEMS IN VISCO-ACOUSTIC MEDIA AND EVALUATION OF ATTENUATION MODEL UNCERTAINTIES

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Abstract

We consider the inverse problem for the quantitative reconstruction of physical parameters in visco-acoustic media, for the recovery of tissue features and identification of embedded bodies. Attenuation is encoded in wave equations. There exists a zoo of such methods in the literature, which are reviewed in the course of this talk.

Finally, we review inverse problems of reconstruction of parameters in visco-acoustic media. Particular emphasis will be given to study a mismatch of models used in forward and inverse simulations.

This is joint work with Florian Faucher, Inria Bordeaux Sud-Ouest, Université de Pau et des Pays de

MINISYMPOSIUM
M20: Inverse Obstacle and Control Problems in Mechanics

Organizers:
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Variational approaches are widely used in all fields of inverse and related ill-posed problems, nonsmooth optimization, optimal control, and homogenization. In this respect, we focus but are not limited to singular and unilaterally constrained problems arising in mechanics, geophysics, and real-world kinetics which are governed by complex systems of PDE equations and inequalities. In a broad scope, the minisymposium objectives are directed toward advances which are attained in the theory, computation, and application of inverse and ill-posed problems.

A NUMERICAL METHOD FOR SOURCE IDENTIFICATION PROBLEM RELATED TO DYNAMICAL KIRCHOFF PLATE EQUATION

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Abstract

Kirchoff Plate model is an integral part of most engineering fields including plate and shell structures [1]. In [2], some important identification problems are stated and some properties are analyzed such as stability and uniqueness. In this work we study the inverse problem of identifying the unknown load distribution $f(x,y)$ in the rectangular domain $\Omega := (0,k) \times (0,l)$ such that

$$\begin{cases} u_{tt} + D\Delta^2 u = g(t) f(x,y) \text{ in } \Omega_t := \Omega \times (0,T) \\ u(x,y,0) = 0, u_t(x,y,0) = 0, \text{ for } (x,y) \in \Omega, \\ u = 0, \partial_n = 0, \text{ on } \overline{\partial\Omega_i} \times [0,T], \text{ for } i = 1,2,3,4, \\ u = 0, -D(\nu u_{xx} + u_{yy}) = 0 \text{ on } \partial\Omega_1 \times [0,T]. \end{cases}$$

Here $u(x,y,t)$ or $(u(x,y,t;f))$ is the displacement at a point $(x,y) \in \overline{\Omega}$ and a time $t \in [0,T]$, $\partial_n u$ denotes the normal derivative of u, $g \in L^2(0,T)$ is the (known) temporal load, $D := E/(1-\nu^2)$ is the bending stiffness, $\nu \in (0,1)$ is the Poisson's ratio, E is the elasticity modulus and $\partial\Omega = \sum_{i=1}^{4} \partial\Omega_i$ where
$\partial\Omega_1 = (0,k) \times \{0\}, \partial\Omega_2 = \{k\} \times (0,l), \partial\Omega_3 = (0,k) \times \{l\}, \partial\Omega_4 = \{0\} \times (0,l)$.
In determination of f we have the following boundary observation on $\overline{\partial\Omega_1}$:
$\theta(x,t) := u_y(x,0,t)$, for $x \in [0,l], t \in [0,T]$

The conjugate gradient algorithm (CGA) is designed for the numerical solution of the identification problem. The proposed approach is based on weak solution theory for PDEs, Tikhonov regularization combined with the adjoint method. Computational results, obtained for noisy output data, are illustrated to show an efficiency and accuracy of the proposed approach, for typical classes of source functions.

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HIGH RESOLUTION INVERSE OBSTACLE SCATTERING USING MULTIPLE FREQUENCY DATA

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Abstract

The problem of recovering the sound profile of a penetrable medium represented by a compactly supported function $q(x)$ given measurements of the far field generated by the scattering of plane waves from multiple known angles of incidence and multiple known frequencies can be cast as

$$F(q)(x) = u^{scat}(x)$$

with F and $u^{scat}(x)$ denoting the forward scattering operator and scattered field, respectively.

To calculate the value of the scattered field u^{scat} off of $q(x)$ given an incident plane wave uinc $u^{scat} = e^{ikx \cdot d}$ for $x \in R^2$ one must solve the variable-coefficient Helmholtz equation
$\Delta u^{scat}(x) + k^2(1+q(x))u^{scat}(x) = -k^2 q(x) u^{inc}(x)$ in $x \in R^2 \setminus \overline{D}$,
together with the Sommerfeld radiation condition. We use the HPS fast direct solver for the Helmholtz equation [4]. The computational cost of this solver is $O\left(N^{3/2}\right)$for the factorization step and $O\left(N\right)$for each new right-hand-side, where N is the number of points used to discretize the domain.

The inverse problem of finding an approximation $\tilde{q}$ of the unknown contrast continuous function $q(x)$ given measurements of the far field u^{scat}scattered by a collection of incident plane waves, can be casted as the nonlinear optimization problem

$$\tilde{q} = \operatorname{argmin}\left\| u^{scat}(x) - F(q) \right\|. \qquad (1)$$

We provide a fast, stable algorithm for the solution of this problem that consists of higher-dimensional extensions of the techniques used in [1]. Problem (1) is both nonlinear and ill-posed. We address nonlinearity through the use of Gauss-Newton's iteration for the frequency, employing the Fréchet derivative of F with respect to q . Despite the fact that the underlying optimization problem is formally ill posed and non-convex, our technique requires only the solution of a sequence of linear least-squares problems at successively higher frequencies, known as the recursive linearization algorithm [3]. By seeking a suitably band-limited approximation of the sound speed profile, each least-squares calculation is well conditioned and involves the solution of a large number of increasingly computationally complex direct scattering problems [2].

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AN INVERSE OBSTACLE PROBLEM FOR THE TIME-DEPENDENT HEAT EQUATION

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Abstract

In this work we deal with the inverse problem of reconstructing the interior boundary curve of a doubly-connected bounded domain given the temperature and the thermal flux on the known exterior boundary curve. We reduce the time-dependent problem to a sequence of stationary problems by applying the Laguerre transform with respect to the time variable. We represent the solution as a modified single-layer potential whose densities have to satisfy a sequence of systems of non-linear boundary integral equations. An iterative algorithm is derived for the numerical solution of the system. We linearize the system using the Fréchet derivative of the corresponding integral operator and we apply Tikhonov regularization to handle its ill-posedness. The numerical results show the feasibility of the proposed scheme.

This is a joint work with R. Chapko (Ivan Franko National University of Lviv, Ukraine).

IMPLEMENTATION OF THE ENCLOSURE METHOD FOR SOME INVERSE CRACK PROBLEMS

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Abstract

An algorithm is introduced for using electrical surface measurements to detect and monitor cracks inside a two-dimensional conductive body. The technique is based on transforming the probing functions of the classical enclosure method by the Kelvin transform. The transform makes it possible to use virtual discs for probing the interior of the body using electric measurements performed on a flat surface.

Theoretical results are presented to enable probing of the full domain to create a profile indicating cracks in the domain.

Feasibility of the method is demonstrated with a simulated model of attaching metal sheets together by resistance spot welding.

This is joint work with Masaru Ikehata, Hiromichi Itou, and Samuli Siltanen

UNIQUENESS IN LOAD IDENTIFICATION IN VIBRATING NANOPLATES

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Abstract

Nanoplates present numerous applications in micro and nano-electromechanical systems as sensors for mass, energy harvesters, or for vibration control. Recently, applications such as the measurement of forces exerted by cells or the intraocular pressure have been proposed in the literature. As far as we know, it is the first time that the uniqueness in the inverse problem m identifying pressure loads acting over a nanoplate from the measurement of its dynamic response is treated. We considered the general case in which a finite number of sources is present, that is, for loads of the type $\sum_{m=1}^{M} g_m(t) f_m(t),\ M \geq 1$, where the time-dependent functions $\{g_m(t)\}_{m=1}^{M}$ are assumed to be known and linearly independent on any sub-interval of the registration time interval. We prove that if the transverse displacement of the nanoplate is known in an open subset compactly contained in the plate domain for an arbitrarily small time interval, then it is possible to identify uniquely the
loads $\{f_m(x)\}_{m=1}^{M}$. Furthermore, we proposed a new method that retakes the idea of using spherical means that can be used in other contexts.

The presentation may interest Engineers, Mathematicians and Physicists working on dynamical problems involving nanostructures and related inverse problems. It is based on the work carried out in conjunction with Antonino Morassi and Ramón Zaera [1].

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MATHEMATICAL MODEL OF CRACK DIAGNOSIS

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Abstract

In this contribution, an optimization technique for identification of single defects like cracks [3] is presented. The respective inverse problem is aimed at crack diagnosis motivated by applications to non-destructive testing with acoustic, electromagnetic, and elastic waves in engineering sciences.
The inverse acoustic scattering model for crack diagnosis is described by Helmholtz problem within mathematic framework and investigated for the sake of scientific computing. Minimizing the misfit from given measurements leads to an optimality condition based imaging function. It is used for non-iterative identification of the center of an unknown crack put in a test domain and characterized by complex-valued surface impedance as suggested in [5]. Further the approach is extended in [1] to the Lippmann-Schwinger equation describing small inhomogeneities by the mean of complex-valued refractive index. In [4] the numerical tests are presented for a cluster of cracks of T-junction shape and are carried out based on the Petrov-Galerkin generalized FEM using local wavelets and level-sets [6]. This shows high-precision identification result and stability to noisy data of the diagnosis, which is illustrated for sound-soft as well as moderately sound-hard cracks when varying the coefficient of surface impedance.
The shape optimization approach is developed in [7] for overdetermined and state-constrained optimization problems of tracking type, and in [2] for identification of breaking lines (a free discontinuity allowing jumps). The corresponding asymptotic analysis realizes on methods of regular as well as singular perturbations.
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ON HOMOGENIZATION OF PERIODIC HYPERBOLIC SYSTEMS

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Abstract

This talk is devoted to homogenization of solutions of periodic hyperbolic systems with rapidly oscillating coefficients. Classical results in homogenization theory look as the convergence of solutions of the problem with rapidly oscillating coefficients to the solution of the so-called effective problem with constant coefficients. The constants in the corresponding error estimates depend on the differential operator, the lattice of periodicity, and the initial data somehow.

We are interested in the operator error estimates. In such estimates, dependence on the initial data in the error estimates is explicit: we have the norm of the data in the error estimate.

STEKLOV AND MODIFIED TRANSMISSION EIGENVALUES AS TARGET SIGNATURES IN AN INVERSE FLUID-SOLID INTERACTION PROBLEM

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Abstract

We investigate an inverse scattering problem for the interaction of waves in a linearly elastic solid and an acoustic fluid by using two sets of eigenvalues: Steklov eigenvalues and modified transmission eigenvalues. For both sets of eigenvalues, we first define them for the problem at hand. We then characterize them by means of operators suitable for the analysis of their location in the complex plane. We also discuss their approximation using far field scattering data via suitable modified far field equations. We finally provide numerical examples that show their approximation from synthetic far field measurements and their application as target signatures.

This is joint work with M. Levitin (University of Reading) and P. Monk (University of Delaware).

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OPTIMIZATION APPROACH IN 2D PROBLEMS OF STATIC FIELDS CLOAKING

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Abstract

In the last few years, devices for cloaking material objects have attracted the rapid attention in the research fields of invisibility and metamaterials. The cloaking scheme of static magnetic fields based on the transformation optics (TO) was proposed in [1]. This scheme using explicit solution possesses several drawbacks. In particular, some components of tensor are singular on the inner boundary of the cloak. Further, another scheme for design of a magnetic cloaking device which is free from the above-mentioned drawback was described in [2]. But it provides only approximate solutions which cannot be implemented because absence required materials in the nature. One of approaches of overcoming mentioned difficulties consists of using the optimization method for solving inverse problems [3, 4].

In this paper the results obtained by using the optimization method to solve cloaking problems with respect to static magnetic fields are present. The theoretical and numerical aspects of this method are discussed in two paper's parts, respectively.

The first part formulates cloaking problem for a 2D model of magnetic scattering. Cloaking effect is achieved by choosing functional parameters of an inhomogeneous anisotropic, in the general case, medium that fills the cloak. Using the method proposed in [3] the problem under consideration is reduced to minimization of a certain cost functional depending on desired medium parameters. The solvability of control problem under study is proved, the optimality system which describes the necessary conditions of extremum is derived, and based on its analysis stability estimates of optimal solutions are established.

In the second part, an effective numerical algorithm is developed for solving problems of magnetic cloaking and shielding based on the use of the particle swarm optimization method (see details in [5]). Important properties of optimal solutions are established. Based on these properties simple design rules are formulated allowing to design cloaking and shielding shells, which possess the high cloaking or shielding performance in addition to easy technical realization.

This work was supported by the Russian Foundation for Basic Research (project no 19-31-90039).

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ELASTOPLASTIC MODELLING AND EXPERIMENTS FOR INVERSE PROBLEMS BASED ON ENTROPY APPROACHES

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Abstract

I will present a novel method to inverse elastoplastic deformation issues based on a recently created concept termed entropy-of-resilience. The Born-Oppenheimer approximation and entropy theory were used to develop non-linear elastoplastic equations for the measured stress and strain curve. Furthermore, by bridging the material's wetting entropy and the system's strain entropy, a relationship between water-contact-angle and elastoplastic strain is established, which can be used to inversely determine the stress-strain plots in elastoplastic deformation using non-destructive water contact angle measurements. Finally, their use in inversely solving elastoplastic problems of sand core production for metal casting and corrosion-reduced mechanical strength of magnesium alloys will be highlighted.

This research was supported by the Agency for Science, Technology and Research (A*STAR) (Singapore), under AME Individual Research Grant (Award No. A20E7c0108); the Ministry of Education (Singapore), under Tier 2 program (Award No. MOE2018-T2-1-163); and the National Research Foundation, Prime Minister's Office (Singapore), under its Marine Science Research and Development program [Award No. MSRDP-P28].

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MINISYMPOSIUM
M21: Imaging Modalities: Recent Advances and Beyond

Organizer:
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The past century has witnessed accelerated development in imaging techniques for medical, biological, industrial and geophysical applications using a wide range of physical modalities. Examples include microscopy, ultrasound, X-ray transmission, positron emission, magnetic resonance, electrical impedance, photoacoustic effect, microwave radiation, atmospheric muons, and seismic waves. The realm of imaging modalities is constantly expanding. Because of their non-intrusive nature, each image reconstruction technique requires finding the solution of an ill-posed mathematical inverse problem to recover the physical properties of a medium using only external measurements. However, each modality and application considered still provides different challenges from both the theoretical and computational point of views. This mini-symposium is aiming to highlight some of the most recent, promising and exciting scientific developments to overcome them.

ULTRASOUND DATA AS A PRIOR IN THORACIC IMAGING WITH ELECTRICAL IMPEDANCE TOMOGRAPHY

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Abstract

Electrical impedance tomography (EIT) has many potential thoracic imaging applications, but images suffer from low spatial resolution due to the severe ill-posedness of the inverse problem. On the other hand, ultrasound tomography (UST) has only been minimally studied for use in thoracic imaging, due to difficulties with wave propagation in lung tissue. However, recent work has indicated low-frequency UST has potential for use in thoracic imaging, while recent advances in direct D-bar reconstruction methods in EIT have led to stabilized reconstructions with improved resolution. In this talk, we use low-frequency UST data to create a spatial prior for direct D-bar EIT reconstruction, thus combining the two modalities.

This is a joint work with Diego Armando Cardona Cárdenas, Sérgio Shiguemi Furuie, and Jennifer L. Mueller

REGULARIZATION OF INVERSE PROBLEMS BY FILTERED DIAGONAL FRAME DECOMPOSITION

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Abstract

The characteristic feature of inverse problems is their instability with respect to data perturbations. In order to stabilize the inversion process, regularization methods have to be developed and applied. We introduce and analyze the concept of filtered diagonal frame decomposition which extends the standard filtered singular value decomposition to the frame case. Frames as generalized singular system allows to better adapt to a given class of potential solutions. We show that filtered diagonal frame decomposition yield a convergent regularization method. Further, we derive convergence rates under source type conditions and prove order optimality under the assumption that the considered frame is a Riesz-basis. In addition, we apply our results to a tomography problem based on the Radon tranform.

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SIMULTANEOUS RECONSTRUCTION OF EMISSON AND ATTENUATION IN PASSIVE GAMMA EMISSON TMOGRAPHY OF SPENT NUCLEAR FUEL

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Abstract

The International Atomic Energy Agency (IAEA) has recently approved passive gamma emission tomography (PGET) as a method for inspecting spent nuclear fuel assemblies (SFAs), an important aspect of international nuclear safeguards. The PGET instrument resembles a single photon emission computed tomography (SPECT) system that allows the reconstruction of axial cross-sections of the emission map of the SFA. The fuel material strongly self-attenuates its gamma-ray emissions, so that correctly accounting for the attenuation is a critical factor in producing accurate images. Due to the nature of the inspections, it is desirable to use as little a priori information as possible about the fuel, including the attenuation map, in the reconstruction process. Current reconstruction methods either do not correct for attenuation, assume a uniform attenuation throughout the fuel assembly, or assume an attenuation map based on an initial filtered back projection (FBP) reconstruction.

We propose a method to simultaneously reconstruct the emission and attenuation maps by formulating the reconstruction as a constrained minimization problem with a least squares data fidelity term and regularization terms. The performance of the proposed method, with two different regularizers, is evaluated with simulated and real data that include missing rods and rods replaced with fresh fuel. The method is shown to produce good results when comparing the reconstructions to the ground truth by various numerical metrics, and when classifying the rods with the method currently employed by the IAEA. Additionally, the proposed method is shown to allow for an enhanced classification method that uses also the reconstructed attenuation map.

This is a joint work with R. Backholm, C. Belanger-Champagne, P. Dendooven, T. Helin and S. Siltanen.

DEEP IMAGE PRIOR RECONSTRUCTION FOR 3D MAGNETIC PARTICLE IMAGING

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Abstract

Magnetic particle imaging (MPI) is a tracer-based imaging modality developed to detect the concentration of superparamagnetic iron oxide nanoparticles. The forward operator describing the relationship between the tracer concentration and the induced voltage in receive coil units is typically given by a Fredholm integral operator of the first kind [1]. A spatial encoding in the time-dependent voltage measurement is obtained by the combination of the nonlinear magnetization behavior of the nanoparticles and the characteristic of the applied magnetic field comprising a static gradient field and a highly dynamic oscillating field. The image reconstruction problem in MPI requires the solution of a severely ill-posed problem [2].

Deep image priors (DIP) have been recently introduced as a machine learning approach for tasks in image processing [3] and its applications to inverse problems [4]. The number of deep learning methods which are applicable to solve inverse problems is continuously increasing. Commonly these methods follow a to step procedure. First a neural network is trained by minimizing a certain loss function for a large number of data points and afterwards new data is fed into the network to obtain a solution to the desired task. In contrast, the DIP approach is different. A solution is obtained by unsupervised training with one single data point. Given one noisy data point y^δ and an ill-posed operator equation $Ax \cong y^\delta$, the task of DIP is to train a network $\varphi_\theta(z)$ with network parameters θ and input z by minimizing

$$\left\| A\varphi_\theta(z) - y^\delta \right\|_1 ,$$

where the random input z is kept fixed. After training the solution to the inverse problem is obtained by evaluating the network, i.e., $x = \varphi_\theta(z)$. Note that the standard DIP uses an l_2-norm in the loss function. For MPI an l1-norm turned out to be beneficial for the reconstruction quality.

In this talk we discuss regularization properties of the DIP method in a specific theoretical setup. Furthermore, we apply a 3D DIP to MPI and show 3D reconstructions from real MPI data. The DIP approach is compared to various regularization methods including the minimization of Tikhonov-type functionals with either l_2-discrepancy or l_1--discrepancy terms. Penalty terms include an l_2-term, an l_1-term, and a TV-term. All methods are also compared to the standard reconstruction technique in MPI, i.e., minimizing a standard Tikhonov functional with a Kaczmarz-type algorithm which in addition includes an implicit regularization mechanism due to early stopping. The latter mechanism turned out to be beneficial for dealing with the noise characteristic in MPI. The behavior of all considered methods is illustrated by quantitative (in terms of image quality; PSNR and SSIM) and qualitative results on a publicly available 3D dataset (Open MPI dataset).

This talk is based on joint work with S. Dittmer, D. Otero Baguer, and P. Maass..

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GEOPHYSICAL AND MEDICAL IMAGING: WHAT THEY CAN LEARN FROM EACH OTHER

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Abstract

Tomographic imaging problems in geophysics and medicine have a lot of common aspects. The at-first-sight similarities are obvious: we have ill-posed inverse problems with an unknown function on an approximately ball-shaped domain. However, there are also remarkable connections in the details. We will investigate this for two inverse problems: the inversion of electro-magnetoencephalography (E-MEG) data in medicine and the inversion of gravitational observations in geophysics. For example, the representation of these inverse problems as Fredholm integral equations of the first kind contain integral kernels which have expansions in terms of Legendre polynomials with very similar coefficients. As a consequence, the null spaces have many common characteristics. Moreover, the well-known downward continuation problem of satellite data also contributes to the exponential ill-posedness of E-MEG inversion.

The speaker and his coauthors have shown that these connections can be utilized to extend the theory of these inverse problems and to develop advanced and particularly adapted numerical methods. We were able to obtain e.g. previously unknown singular value decompositions and insights on the ill-posedness of the inverse problems. Moreover, our numerical approaches managed to outperform some previously known methods. This was verified with synthetic, but realistic data examples. Moreover, we also demonstrated the applicability to real data.

This talk is an attempt to give an overview on some of the results which were obtained on inverse gravimetry (and the related downward continuation) and on E-MEG inversion.

Parts of this presentation are based on joint works with A.S. Fokas, O. Hauk, S. Leweke, and R. Telschow. For further details, see [1-10].

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MINISYMPOSIUM
M22: Inverse Problems with Data-Driven Methods and Deep Learning

Organizers:
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We are currently experiencing a paradigm shift in image reconstruction. Robust mathematical inversion algorithms are combined with emerging methods in data science to achieve state-of-the-art results in a wide range of inverse problems. A successful application of these methods in practice involves a thorough understanding of their mathematical properties and the underlying imaging physics. This minisymposium aims to bring experts in data-driven methods and deep learning for inverse problems together and provides an overview of learned image reconstruction approaches, mathematical insights, and real-world applications.

STILL NO FREE LUNCH – ON AI GENERATED HALLUCINATIONS AND THE ACCURACY-STABILITY TRADE-OFF IN INVERSE PROBLEMS

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Abstract

AI generated hallucinations are becoming a serious concern in modern methods for image reconstruction in inverse problem. The phenomenon seems universal and has become apparent in a variety of modalities:

"The most serious issue when applying deep learning for discovery is that of hallucination. [...] These hallucinations are deceptive artifacts that appear highly plausible in the absence of contradictory information and can be challenging, if not impossible, to detect."

– In "Applications, promises, and pitfalls of deep learning for fluorescence image reconstruction", Nature Methods (2021) [1].

"The potential lack of generalization of deep learning-based reconstruction methods as well as their innate unstable nature may cause false structures to appear in the reconstructed image that are absent in the object being imaged "

– In "On hallucinations in tomographic image reconstruction", IEEE Trans. Med. Imaging (2021) [2].

In this talk, we present a comprehensive mathematical analysis explaining the many facets of AI generated hallucinations, their links to instabilities, but also how stable AI methods can hallucinate. Our results establish four crucial issues for AI methods in inverse problems that can be interpreted as 'no free-lunch' phenomena: (1) Overly accurate AI methods will wrongly transfer details from one image to another reconstructed image creating a hallucination. Our mathematical theory describes exactly how these details very naturally can appear in data-driven setups and why this phenomenon occurs. (2) There is an accuracy-hallucination barrier: Too accurate reconstructions – on for example the training set – will yield hallucinations, and reducing the accuracy on certain inputs will reduce the hallucinations. (3) There is an accuracy-stability trade-off, and optimising this trade-off is challenging through standard training processes. (4) Hallucinations can occur due to any kind of noise model and probability distribution on the data used, and standard training has few ways of protecting against the conditions leading to AI generated hallucinations.

Finally, our results show that accurate and hallucination-free methods can only be achieved by having information about the kernel of the sampling operator encoded in the recovery algorithm. Based on this, we initiate a program for reducing hallucinations in DL in inverse problems. This suggests a path forward toward hallucination free AI methods in inverse problems.

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A BAYESIAN DEEP IMAGE PRIOR

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Abstract

Existing deep learning based tomographic image reconstruction methods do not provide accurate estimates of reconstruction uncertainty, hindering their real-world deployment. To address this limitation, we construct a Bayesian prior for tomographic reconstruction, which combines the classical total variation (TV) regulariser with the modern deep image prior (DIP). Specifically, we use a change of variables to connect our prior beliefs on the image TV semi-norm with the hyper-parameters of the DIP network. For the inference, we develop an approach based on the linearised Laplace method, which is scalable to high-dimensional settings. The resulting framework provides pixel-wise uncertainty estimates and a marginal likelihood objective for hyperparameter optimisation. We demonstrate the method on synthetic and real-measured high-resolution μCT data, and show that it provides superior calibration of uncertainty estimates relative to previous probabilistic formulations of the DIP.

This is joint work with J Antorán, J Leuschner, JM Hernández-Lobato, B Jin

GROUND TRUTH FREE DENOISING BY OPTIMAL TRANSPORT

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Abstract

This study proposes a new training strategy for a denoiser removing (additive) independent noise, with only as readily available data as possible and no further assumptions on the data nor noise. While every real-world measurement contains some noise, it seems that this problem remains unsolved for settings where clean data samples are lacking.

We propose a pushforward operator formulation of an ideal denoiser and a corresponding GAN setup for training a denoiser ground truth free. The GAN trains solely on samples of noisy data and noise. In a series of denoising experiments in 1D and 2D, we demonstrate our training strategy's performance, which significantly improves the state-of-the-art of unsupervised denoising. Moreover, for some non-Gaussian noise, the method compares favorably even to naive supervised denoising.

REGULARISING INVERSE IMAGING PROBLEMS USING GENERATIVE DEEP LEARNING METHODS

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Abstract

Deep neural network approaches to inverse imaging problems have produced impressive results in the last few years. We consider the use of generative models in a variational regularisation approach to inverse problems. Generative models learn, from observations, approximations to high-dimensional data distributions. The considered regularisers penalise images that are far from the range of a generative model that has learned to produce images similar to a training dataset. We name this family *generative regularisers.*

For example, let A:$X\rightarrow Y$ be a forward process that takes an image $x\in X$ to data $y\in Y$. The inverse problem takes data, y corrupted by noise, ϵ, and finds image, x, such that y=$A(x)$. A trained generator G:$Z\rightarrow X$, takes values in a known lower dimensional latent space, Z and outputs images similar to some training set. One common approach to incorporate the generator as part of a regulariser, introduced by Bora et al. [1], searches through the range of the generator to solve the inverse problem:

$$z^* \in \arg\min_{z\in Z} \left\| A(G(z)) - y \right\|,\ x^* = G(z^*) \qquad (1)$$

Similar ideas have been further explored by [2,3,4].

In contrast to other data driven approaches, generative regularisers do not require paired training data and are learnt independently of the forward model. This makes the method very flexible in real-world scenarios where noise levels and forward model parameters may change.

In this talk, we will give numerical examples, using variational autoencoders (VAEs) [5] and generative adversarial networks (GANs) [6] demonstrating three different approaches of including generative models in solving inverse problems on simple datasets. The success of these methods depends on the quality of a generator. For example, in a deterministic setting, such as (1), we also require that the ground truth image lies in the range of the generator. We will discuss desired criteria for evaluating a trained generative network, in the context of inverse problems, allowing comparison between generative model approaches and providing direction for future work.

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EFFICIENT TRAINING OF INFINITE-DEPTHE NEURAL NETWORKS VIA JACOBIAN-FREE BACKPROPAGATION

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Abstract

A promising trend in deep learning replaces fixed depth models by approximations of the limit as network depth approaches infinity. This approach uses a portion of network weights to prescribe behavior by defining a limit condition. This makes network depth implicit, varying based on the provided data and an error tolerance. Moreover, existing implicit models can be implemented and trained with fixed memory costs in exchange for additional computational costs. In particular, backpropagation through implicit networks requires solving a Jacobian-based equation arising from the implicit function theorem. We propose a new Jacobian-free backpropagation (JFB) scheme that circumvents the need to solve Jacobian-based equations while maintaining fixed memory costs. This makes implicit depth models much cheaper to train and easy to implement. Numerical experiments on classification and medical imaging are provided.

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MEASURING AND ENHANCING ROBUSTNESS IN DEEP LEARNING BASED COMPRESSIVE SENSING

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Abstract

Deep-learning based algorithms outperform traditional, handcrafted algorithms for reconstructing images from few and noisy measurements. However, neural networks may be sensitive to small, yet adversarially-selected perturbations, may perform poorly under distribution shifts, and may fail to recover small but important features in an image.

To understand the sensitivity to such perturbations, we measured the robustness of a variety of deep network based and traditional methods. Perhaps surprisingly, in the context of accelerated magnetic resonance imaging, we find no evidence that deep learning based algorithms are less robust than classical, un-trained methods. Even for natural distribution shifts, we find that classical algorithms with a single hyper-parameter tuned on a training set compromise as much in performance than a neural network with 50 million parameters. Our results indicate that the state-of-the-art deep-learning-based image reconstruction methods provide improved performance than traditional methods without compromising robustness.

Finally, we present a test-time-training method to improve the distributional robustness of deep-learning based imaging.

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STOCHASTIC NORMALIZING FLOWS FOR INVERSE PROBLEMS: A MARKOV CHAINS VIEWPOINT

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Abstract

Normalizing flows are invertible neural networks which aim to learn a diffeomorphism pushing forward the normal distribution onto a more complicated distribution. They can be used for generative modeling and for estimating probability density functions from given samples. However, recently it was shown that learning multimodal or heavy tailed distributions with normalizing flows requires exploding Lipschitz constants. Thus, using normalizing flows for generative modeling requires large architectures and the training suffers from instabilities.

To overcome these topological constraints and to improve the expressiveness of normalizing flow architectures, Wu, Köhler and Noé introduced stochastic normalizing flows which combine deterministic, learnable flow transformations with stochastic sampling methods. In this talk, we consider stochastic normalizing flows from a Markov chain point of view. In particular, we replace transition densities by general Markov kernels and which allows to incorporate distributions without densities in a sound way. Here, we derive a loss function based on the Kullback-Leibler divergence and Radon Nikodym derivatives. Further, we show that stochastic normalizing flows build a unifying framework for a wide range of powerful generative models as variational autoencoders and diffusion normalizing flows. Thus, our framework establishes a useful mathematical tool to combine the various approaches. For tackeling Bayesian inverse problems, we extend the ideas of conditional and stohastic normalizing flows for sampling from the posterior distribution.

Finally, we demonstrate the performance of stochastic normalizing flows for some numerical examples. This is a joint work with Paul Hagemann and Gabriele Steidl.

SOLVING MMV PROBLEMS VIA ALGORITHM UNFOLDING

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Abstract

Recovering structured data from compressive observations is a core task many fields, like computational imaging, communications and information processing. These problems are ill-posed and prior assumptions are necessary to restrict the solutions.

A particular instance is the multiple measurement vector (MMV) problem where one wants to sense joint-sparse vectors from a few compressive and repeated observations, far less than its ambient dimension. Fundamental works show that under certain conditions this can be provably achieved in a robust and stable manner with computationally tractable algorithms.

For real-world problems, however, it is difficult and often impossible to analytically treat detailed structure beyond joint-sparsity and to perform optimal algorithm tuning. Therefore, recovery approaches, well-understood in theory, perform often suboptimal in practice. Algorithms converge slowly and increased acquisition time and sampling rates are necessary to achieve a given target resolution. Approaching high-dimensional applications under almost real-time conditions is therefore often not feasible.

In this talk I will present joint works with J. Hauffen and O. Musa on recent unfolding approaches for the MMV problem. Only few iterations of an algorithm are unrolled into a neural network and the parameters are trained data-driven via gradient descent and back-propagation. However, learning all weights in a high-dimensional setting requires a large amount of training data and in many application training data is scarce.

Therefore, I will discuss here the case of analytic weight computation which tremendously reduce training overhead and makes this approach more accessible to many new applications.
Furthermore, as in the single vector case, this even permits a theoretical treatment of the trained architecture. The results will be more refined to convolutional models and then discussed in the context applications in photo-thermal imaging and in wireless communication.

RECOVERY ANALYSIS FOR PLUG-AND-PLAY PRIORS USING THE RESTRICTED EIGENVALUE CONDITION

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Abstract

The plug-and-play priors (PnP) and regularization by denoising (RED) methods have become widely used for solving inverse problems by leveraging pre-trained deep denoisers as image priors. While the empirical imaging performance and the theoretical convergence properties of these algorithms have been widely investigated, their recovery properties have not previously been theoretically analyzed. We address this gap by showing how to establish theoretical recovery guarantees for PnP/RED by assuming that the solution of these methods lies near the fixed-points of a deep neural network. We also present numerical results comparing the recovery performance of PnP/RED in compressive sensing against that of recent compressive sensing algorithms based on generative models. Our numerical results suggest that PnP with a pre-trained artifact removal network provides significantly better results compared to the existing state-of-the-art methods.

2DETECT - A LARGE 2D EXPANDABLE, TRAINABLE, EXPERIMENTAL COMPUTED TOMOGRAPHY DATA COLLECTION FOR MACHINE LEARNING

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Abstract

Recent research in the computational imaging community largely focuses on the use of machine learning (ML) techniques for tomographic image reconstruction. Realistic experimental data, i.e. suitable projection datasets with high-quality ground truth reconstructions and/or segmentations are scarce, but highly important for the development and training of deep learning algorithms. Our open data collection fills this gap and provides the community with a versatile 2D CT dataset for machine learning. For this, a diverse mix of samples with high natural variability in both, inter and intra-sample shape and density, were scanned with a laboratory X-ray set-up. This sample mix of dried fruits and nuts was filled in a carboard tube (height 34cm, diameter 10cm) and scanned slice by slice in a fan-beam acquisition setting. More than 5000 slices consisting of 3600 projections per full orbit were scanned in two beam settings, reconstructed with highest quality, and supplemented with a ground truth segmentation. We provide the complete image reconstruction pipeline: raw projection data (sinograms), a description of the scanning geometry, pre-processing and reconstruction scripts using open software, and the reconstructed slices as well as their ground truth segmentations. Due to this, the dataset can be used not only for ML-based reconstruction or segmentation in various settings such as limited or sparse angle, but also for noise reduction for general algorithm development.

SOLVING INVERSE PROBLEMS WITH DEEP NEURAL NETWORKS - ROBUSTNESS INCLUDED

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Abstract

In the past five years, deep learning methods have become state-of-the-art in solving various inverse problems. Before such approaches can _nd application in safety-critical fields, a verification of their reliability appears mandatory. Recent works have pointed out instabilities of deep neural networks for several image reconstruction tasks. In analogy to adversarial attacks in classification, it was shown that slight distortions in the input domain may cause severe artifacts. We shed new light on this concern, by presenting an extensive study of the robustness of deep-learning-based algorithms for solving underdetermined inverse problems. This covers compressed sensing with Gaussian measurements as well as image recovery from Fourier and Radon measurements, including a real-world scenario for magnetic resonance imaging (using the NYU-fastMRI dataset). Our main focus is on computing adversarial perturbations of the measurements that maximize the reconstruction error. A distinctive feature of our approach is the quantitative and qualitative comparison with total-variation minimization, which serves as a provably robust reference method. In contrast to previous findings, our results reveal that standard end-to-end network architectures are not only resilient against statistical noise, but also against adversarial perturbations.

All considered networks are trained by common deep learning techniques, without sophisticated defense strategies.

This is joint work with Martin Genzel and Maximilian Marz.

DATA-DRIVEN ADVERSARIAL REGULARIZATION FOR IMAGING INVERSE PROBLEMS

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Abstract

Variational reconstruction has by far been the most widely used approach for solving ill-posed inverse problems in imaging applications. Nevertheless, the impressive success of deep learning in recent layers has led to the development of a number of data-driven methods that significantly outperform the classical variational approaches in terms of reconstruction quality. Such data-driven approaches, although numerically superior to the classical variational approaches with hand-crafted regularizers, generally come with no theoretical stability and convergence guarantees. Within the realm of learning-based approaches, there exists a particularly notable line of research that seeks to model the regularizer in the variational setting using a deep neural network and learn the parameters using an ensemble of training data.

In this talk, we will introduce some new approaches based on adversarial learning to construct data-driven variational regularizers that are amenable to stability analysis and lead to scalable and provably convergent reconstruction algorithms. More specifically, we will (i) touch upon the problem of learning a convex regularizer using input-convex neural networks and (ii) show how to combine iterative unrolling with data-driven regularization to achieve fast, high-quality, and provably stable reconstruction

LEARNED SPEED OF SOUND CORRECTION FOR PHOTOACOUSTIC TOMOGRAPHY

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Abstract

Photoacoustic tomography is a hybrid imaging modality that combines optical contrast with ultrasonic detection, and it has shown the potential in a variety of biomedical applications. Real-time applications in three-dimensional photoacoustic tomography are relying on fast reconstruction algorithms, instead of utilising full wave solvers. Unfortunately, the majority of available fast solvers assume homogenous speed of sound (SoS) distributions. Additionally, reconstruction quality depends on the correct choice for a constant SoS in the imaged target. In practical experiments, accurate knowledge of the SoS of the imaged target is commonly not available. To overcome these limitations, we propose a data-driven correction to compensate for unknown or heterogeneous SoS distributions. Especially, we evaluate the potential to include a learned model correction in k-space into the fast FFT based reconstruction algorithm in combination with a learned post-processing and compare to post-processing alone.

The feasibility of the approaches is studied with simulated data and human in vivo measurements.

SOLVING ILL-POSED INVERSE PROBLEM WITH PRETRANED DENOISERS, GANS AND SUPER-RESOLVERS: THE BP TERM AND THE CORRECTION FILTER

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Abstract

Ill-posed linear inverse problems appear in many image processing applications, such as deblurring, super-resolution and compressed sensing. Many restoration strategies involve minimizing a cost function, which is composed of fidelity and prior terms, balanced by a regularization parameter. While a vast amount of research has been focused on different prior models, the fidelity term is almost always chosen to be the least squares (LS) objective, that encourages fitting the linearly transformed optimization variable, Ax, to the observations y:

$$\ell_{LS}(x,y) = \frac{1}{2}\left\|y - Ax\right\|_2^2. \quad (1)$$

In this talk, we will propose and examine a different fidelity term, dubbed the back-projection (BP) term, which "back-projects" the residual to the signal space using the (possibly regularized) pseudo-inverse ($A^{\div}$) of the linear operator:

$$\ell_{BP}(x,y) = \frac{1}{2}\left\|A^{\div}(y - Ax)\right\|_2^2. \quad (2)$$

Using the simple proximal gradient method with the BP term and off-the-shelf CNN denoisers or GANs (that impose the prior) gives excellent empirical results and requires less parameter tuning than LS-based methods [1]. Moreover, I will present several theoretical results showing cases with advantages of BP over LS both in accuracy and in convergence rate [2], [3].

While using off-the-shelf CNN denoisers and GANs in inverse problems has recently become extremely popular, it is much less common to use other more "task-specific" networks for observation models that have not been considered in their training phase. For example, state-of-the-art CNN super-resolvers suffer from a huge performance drop when the downscaling kernel in test time even slightly mismatches the kernel that is used for training the networks.

We will show how to mitigate this issue for off-the-shelf CNN super-resolvers, which are (typically) trained with the bicubic downscaling kernel (without any change in the networks). Specifically, inspired by the literature on generalized sampling, we design a correction filter h that modifies the low-resolution image to mimic one which is obtained by another kernel (e.g., the bicubic) [4]. Thus, applying existing pre-trained super-resolvers on the "corrected" low-resolution image, $h*y$, significantly improves their results. We also show that our approach outperforms other super-resolution methods that are designed for general downscaling kernels. h
□hy

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MINISYMPOSIUM
M23: Theory and Numerics for Inversion Strategies

Organizer:
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In recent decades, the inverse problem community has witnessed remarkable developments in both theoretical and numerical aspects. In this mini-symposium, we are aiming to bring together many experts mainly from Asia who are working in a wide range of inverse problems and their related fields concerning some inversion strategies for inverse problems. This will give us a good opportunity to share some motivations, backgrounds and viewpoints of several inverse problems. Further, it will also improve the communication and collaboration especially between researchers in Asia and Europe.

LOCAL RECOVERY OF A PIECEWISE CONSTANT ANISOTROPIC CONDUCTIVITY IN EIT ON DOMAINS WITH EXPOSED CORNERS

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Abstract

We study the local recovery of an unknown piecewise constant anisotropic conductivity in EIT (electric impedance tomography) on certain bounded Lipschitz domains Ω in $\mathbb{R}^2$ with corners. The measurement is conducted on a connected open subset of the boundary $\partial\Omega$ of Ω containing corners and is given as a localized Neumann-to-Dirichlet map. The above unknown conductivity is defined via a decomposition of Ω into polygonal cells. Specifically, we consider a parallelogram-based decomposition and a trapezoid-based decomposition. We assume that the decomposition is known, but the conductivity on each cell is unknown. We prove the local recovery near a known piecewise constant anisotropic conductivity γ_0. We do so by proving the injectivity of the Fr\'echet derivative $F'(\gamma_0)$ of the forward map F, say, at γ_0. The proof presented, here, involves defining different classes of decompositions for γ_0 and a perturbation or contrast H in a proper way so that we can find in the interior of a cell for γ_0 exposed single or double corners of a cell of $\mbox{\rm supp}H$ for the former decomposition and latter decomposition, respectively. Then, by adapting the usual proof near such corners, we establish the mentioned injectivity.

This is a joint work with Maarten de Hoop (Rice University), Ching-Lung Lin (National Cheng-Kung University), Gen Nakamura (Hokkaido University), Manmohan Vashith (Indian Institute of Technology).

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EXTRACTION OF THE MASS DENSITY BY EMBEDDING CONTRASTED SMALL INCLUSIONS

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Abstract

We deal with the time-harmonic elastic wave propagation in the presence of the small-scaled particles of high relative mass density. These particles generate few local spots at their locations and are generated as the possible body waves related to elastic resonances. We can observe that the eigenvalues of the elastic Newtonian potential and a family of the mentioned resonances are related.

Our aim is twofold. First, to characterize the dominant scattered fields due to the injection of small particles for incident frequencies near resonances. Second, to tackle the inverse problem of reconstructing the mass density.

The main idea in achieving our goal is to invert the associated Lippmann Schwinger system by deriving required apriori estimates of the total fields to get the dominant field, which further allows us to extract the mass density. This procedure has advantages in comparison with the classical problem based on the D-N map.

CARLEMAN ESTIMATES AND INVERSE PROBLEMS FOR THE COUPLED QUANTITATIVE THERMO-ACOUSTIC EQUATIONS

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Abstract

We investigate the so called quantitative thermo-acoustic tomography process (e.g. [1, 3, 10, 11] and their references). According to [3], assuming that the variations in temperature and pressure are weak and neglecting the nonlinear effects, we obtain the following system:

$$\begin{cases} \partial_t^2 p - \rho v_s^2 div\left(\frac{1}{\rho}\nabla p\right) - \Gamma\partial_t\{div(\kappa\nabla\theta)\} = \Gamma\partial_t\Pi_\alpha, \\ \partial_t\theta - \frac{1}{\rho C_p} div(\kappa\nabla\theta) - \frac{\theta_0\varsigma}{\rho C_p}\partial_t p = \frac{\Pi_\alpha}{\rho C_p}, \end{cases} \text{ in } Q \qquad (1)$$

for the temperature rise θ and the pressure perturbation p from the equilibrium steady state depending on $(x,t) := (x_1,\ldots,x_n,t) \in Q := \Omega\times(0,T).$

We set $\Theta(x,t) = \partial_t\theta(x,t).$ Differentiating the second equation in (1) with respect to t, we obtain:

$$\begin{cases} \partial_t^2 p - \rho_1(x)div(q(x)\nabla p) - \rho_2(x)div(\kappa(x)\nabla\Theta) = \Gamma(x)\partial_t\Pi_\alpha, \\ \partial_t\Theta - \rho_3(x)div(\kappa(x)\nabla\Theta) - \rho_4(x)\partial_t^2 p = \rho_3(x)\partial_t\Pi_\alpha, \end{cases} \text{ in } Q, \qquad (2)$$

where $\rho_1(x) = \rho(x)v_s^2\rho(x)$, $q(x) = 1/\rho(x)$, $\rho_2(x) = \Gamma(x)$, $\rho_3(x) = 1/(\rho(x))C_p(x)$ and $\rho_4(x) = \theta_0(x)\varsigma(x)\rho_3(x)$.

We will establish Carleman estimates for (2) with the exponential weight function $\exp(2s\varphi)$ where

$$\varphi(x,t) = \exp(\lambda\psi(x,t)),\ \psi(x,t) = |x - x_0|^2 - \tau t^2,\ (x,t)\in Q$$

$\lambda > 0$ is a suitably large constant, and $\tau > 0$ 0 is a suitably small constant, provided that K_θ, β, C_p, θ_0, ρ_0 and κ satisfy some boundedness and positivity conditions. By these Carleman estimates and the Bukhgeim-Klibanov method (e.g., [2, 4, 5, 7, 8, 9] and their references), we will consider several inverse problems for (2).

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DIFFUSE OPTICAL TOMOGRAPHY WITH A SIMULATED ANNEALING MONTE CARLO ALGORITHM

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Abstract

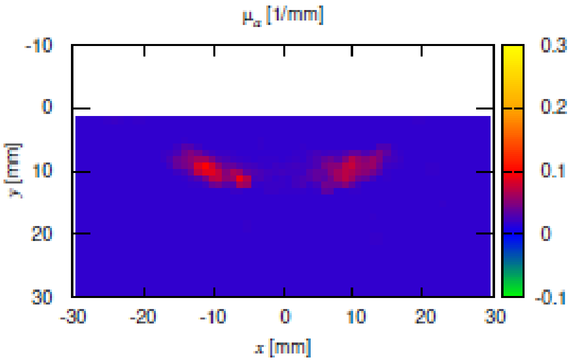
Let us consider the following diffusion equation:

$$\begin{cases} -D_0\Delta u + \mu_0 u = f, & x \in \Omega, \\ \varsigma D_0 \partial_\nu u + u = 0, & x \in \partial\Omega, \end{cases}$$

We will consider the inverse problem of determining $\mu_a(x)$ from boundary measurements.

We propose a Monte Carlo numerical method for this diffuse optical tomography which uses the single-component Metropolis-Hastings algorithm. After a spin Hamiltonian is introduced in the cost function, the inverse problem is solved by simulated annealing.

One of reconstructed images is shown below.



Two absorbers with separation 20 mm.

UNIQUENESS FOR THE INVERSE BOUNDARY VALUE PROBLEM OF PIECEWISE HOMOGENEOUS ANISOTROPIC ELASTICITY IN THE TIME DOMAIN

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Abstract

We consider the inverse boundary value problem of recovering a piecewise homogeneous elastic tensor and a piecewise homogeneous mass density from a localized lateral Dirichlet-to-Neumann or Neumann-to-Dirichlet map, for the elasticity equation in the space-time domain.

We derive uniqueness for identifying this tensor and density on alldomains of homogeneity that may be reached from the part of the boundary where the measurements are taken by a chain of subdomains whose successive interfaces contain a curved portion.

INVERSE SOURCE PROBLEMS FOR WAVE EQUATIONS ON LORENTZIAN MANIFOLDS

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Abstract

One obtains a system of wave equations with time-dependent principal parts by linearizing the Einstein equation. Due to the time dependence, it has not been sufficient to investigate inverse problems for such kinds of hyperbolic equations in Euclidean spaces. Therefore, we introduce geometric analysis on semi-Riemannian manifolds, especially Lorentzian manifolds, and prove conditional local Hölder stability for inverse source problems. I will show Sobolev spaces on manifolds, semigeodesic coordinate systems, and Carleman estimates on manifolds.

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ON THE IDENTIFICATION OF SOURCE TERM IN THE HEAT EQUATION FROM SPARSE DATA

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Abstract

We consider the recovery of a source term $f(x,t) = p(x)q(t)$ for the nonhomogeneous heat equation in $\Omega \times (0,\infty)$ where Ω is a bounded domain in R^2 with smooth boundary $\partial\Omega$ from overposed lateral data on a sparse subset of $\partial\Omega \times (0,\infty)$. Specifically, we shall require a small finite number N of measurement points on $\partial\Omega$ and prove a uniqueness result; namely the recovery of the pair $(p;q)$ within a given class, by a judicious choice of $N = 2$ points. Naturally, with this paucity of overposed data, the problem is severely ill-posed.

Nevertheless we shall show that provided the data

MINISYMPOSIUM
M24: Inverse Problems via Topological Derivatives

Organizer:
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The topological derivative of a shape functional measures the sensitivity of such functional to having an infinitesimal perturbation at each point of the explored region. It has many applications in connection with inverse problems such as shape optimization, topology optimization, imaging processing, crack and defect detection in non-destructive testing, to mention a few.
The aim of this minisymposium is to bring together experts and young researchers working in this field to discuss and review the recent applications, new results and future challenges.

TOPOLOGICAL DERIVATIVE BASED BAYESIAN INFERENCE FOR INVERSE SCATTERING PROBLEMS

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Abstract

Inverse scattering techniques seek to infer the structure of objects integrated in an ambient medium from data recorded at a set of receptors, which represent some scattered incident radiation.
Solving the inverse problem amounts to finding objects minimizing the difference between the synthetic data generated by the approximate objects as predicted by a forward model and the true data. When the magnitude of the noise in the data is small, algorithms combining iteratively regularized Gauss-Newton schemes with topological derivative based initial guesses and updates of the number of objects may provide reasonable reconstructions. However, estimating uncertainties inherent to this process as the magnitude of the noise increases is a challenging task.

We propose a topological derivative based Bayesian inference framework.

Numerical simulations illustrate the resulting predictions in light and acoustic holography set-ups.

RECONSTRUCTION OF SHARP INTERFACES IN TIME-DOMAIN FULL WAVEFORM INVERSION

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Abstract

Most optimization approaches for reconstruction of velocity models used in FWI are based on smooth techniques such as the Tikhonov regularization. However, realistic velocity profiles usually present various discontinuities and sharp interfaces, for instance in the case of salt bodies. Thus, approaches based on nonsmooth optimizations techniques should be used to obtain sharper reconstructions.

In this work, we propose a novel shape optimization based algorithm to track sharp interfaces of the velocity model for time-domain FWI. The interface of a salt domain is considered as the variable of a tracking-type cost functional. A damping term in the neighborhood of the boundary is used to model an unbounded domain. Using a Lagrangian approach, we compute the shape derivative of the cost functional. The shape derivative depends on an adjoint state, which is the solution of a wave equation with the residual on right-hand side. The interface evolution is performed using a level set method. Using synthetic measurements, we show the efficiency of the method through several examples of reconstruction.

TOPOLOGICAL SENSITIVITY ANALYSIS FOR IDENTIFICATION OF VOIDS UNDER NAVIER'S BOUNDARY CONDITIONS IN LINEAR ELASTICITY

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Abstract

This work is concerned with a geometric inverse problem related to the two-dimensional linear elasticity system. Thereby, voids under Navier's boundary conditions are reconstructed from the knowledge of partially over-determined boundary data. The proposed approach is based on the so-called energy-like error functional combined with the topological sensitivity method. The topological derivative of the energy-like misfit functional is computed through the topological-shape sensitivity method. Firstly, the shape derivative of the corresponding misfit function is presented briefly from previous work Méjri (2018 *J. Inverse Ill-Posed Problems*). Then, an explicit solution of the fundamental boundary-value problem in the infinite plane with a circular hole is calculated by the Muskhelishvili formulae. Finally, the asymptotic expansion of the topological gradient is derived explicitly with respect to the nucleation of a void.

Numerical tests are performed in order to point out the efficiency of the developed approach.

DETECTION OF SMALL CARDIAC ISCHEMIC REGIONS FROM BOUNDARY MEASUREMENTS VIA TOPOLOGICAL GRADIENT

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Abstract

The monodomain model is one of the most famous mathematical descriptions of the cardiac electrophysiology, and it consists of a system of a semilinear parabolic partial differential equation with nonlinear ordinary differential equations. Our aim is to detect the presence of small ischemic regions, namely, a perturbation in the coefficients of the model, from boundary data.

After an analysis of the well-posedness of the model we determine an asymptotic expansion of the perturbed potential due to the presence of small conductivity inhomogeneities and use it to compute the topological gradient of a suitable boundary misfit functional.

The stability of the reconstruction, even in the case of noisy measurements, is confirmed by several numerical experiments.

This is joint work with Elena Beretta (Politecnico di Milano – NYU Abu Dhabi) and Cecilia Cavaterra (Università Statale di Milano).

A REAL-TIME IDENTIFICATION OF SMALL CONDUCTIVITY INHOMOGENEITY VIA TOPOLOGICAL DERIVATIVE

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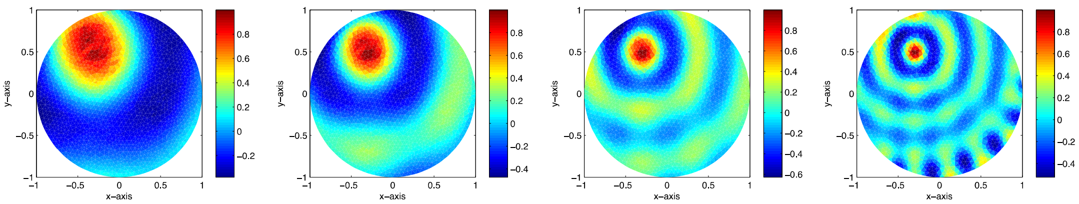
Abstract

There exists a considerable amount of interesti.ng inverse problems concerned with the retrieval of inclusions embedded in a medium from measured boundary data. This is an old but very interesting to nowadays scientists due to its wide applicability such as detection of cracks in material engineering, ultrasound imaging in medical sciences, and scanning anti-personnel mines hidden in the ground in military services. Throughout various researches, remarkable reconstruction techniques have been investigated, most of which are based on the Newton-type iteration scheme. For a successful application of iteration based scheme, the one must begin with an initial guess that is close to the true one in order to avoid undesirable situations such as non-convergence or falling into the local minima. Therefore, development of a mathematical theory and an algorithm for generating a good initial guess is an important research topic.

Topological derivative (TD) concept is a non-iterative imaging strategy. This was originally developed for the shape optimization problem, but its application to rapid shape identification has only recently been proven. Related works can be found in [1,2,3,4] and references therein. Recently, TD was applied to the inverse conductivity problems as an iterative scheme [5,6,7]. Following [8], it has been confirmed that TD can be applied as a non-iterative scheme but an estimation of total number of anomalies is needed.

In this contribution, we apply TD for identifying single, small conductivity inhomogeneity without any a priori information. For this, we design a TD based indicator function with two-different boundary conditions and analyze its mathematical structure. To explain the feasibility and limitation, simulation results with noisy data are exhibited.



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TOPOLOGICAL DERIVATIVE ALGORITHM TESTED AGAINST EXPERIMENTAL DATA: THE THREE-DIMENSIONAL FRESNEL DATABASE

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Abstract

The Fresnel databases [1, 2] were conceived as tools for benchmarking different inversion algorithms, as well as providing experimental data to research groups that lack the facilities or laboratories to generate their own.

They consist of several files with measurements of the electromagnetic field for different electromagnetic experiments. In these experiments, several dielectric or metallic objects were irradiated from different positions and frequencies while the electric field was recorded on a set of antennas.

In [3] an algorithm based on the topological derivative [4] was tested against the Fresnel two- and three-dimensional databases with very good results. The algorithm consists in computing the topological derivative of a functional measuring the difference between the recorded measures and the ones expected when a known object is present. The topological derivative is a field which gives an idea of how sensitive is that functional to the effect of having an inclusion or a hole in each point in the domain.

With this in mind, the points where the topological derivative attains the largest negative values are identified as belonging to the actual objects.

The formulae for the computation of the topological derivative both for the dielectric and the conductive cases were already obtained in [5], where the same algorithm was tested against numerically generated data.

In this talk, the results from [3] will be shown, together with a brief explanation of the algorithm used. We will take special care in describing the results of the three-dimensional database [2] . The processing

of this database was much more demanding as the computational cost was higher and also because the full three-dimensional shape of the objects had to be recovered given only measures taken on the equatorial plane.

Nevertheless, the topological derivative algorithm proved to be a very suitable method for solving this kind of inverse problem, as it is a fast and robust method, with no need of any kind of prior information. In this sense, its performance was comparable to that of the other methods tested against the same database.

Finally we will also comment on several possible improvements to the method, some of them are ongoing work.

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MINISYMPOSIUM
M25: Inverse Problems in Biomedical and Material Imaging

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Imaging techniques are powerful tools for the analysis of human organs, biological systems or mechanical and optical materials. They range from different kinds of tomography such as X-ray CT (Computed Tomography), EIT (Electrical Impedance Tomography), DOT (Diffuse Optical Tomography), PT (Photoacoustic Tomography), OCT (Optical Coherence Tomography), Ultrasound Imaging, Elastography, etc. to different kinds of microscopy (fluorescence microscopy, single-molecule fluorescence, Light Sheet Fluorescence Microscopy, etc.). They are based on the solution of inverse problems that require mathematical modeling of the acquisition process and numerical methods for the solution of the equations relating the acquired data to the unknown object.

The purpose of this mini-symposium is to bring together researchers with different backgrounds and interests in Inverse Problems in Biomedical and Material Imaging. All researchers, academicians, practitioners, as well as students interested in these knowledge areas, are invited to submit their recently achieved results in modeling, solution methods and their use on real data from biomedical applications and non destructive material testing.

DIRECT INVERSION METHOD FOR QUASI-STATIC MEDICAL ELASTOGRAPHY: STABILITY AND DISCRETIZATION

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Abstract

This work focuses on the reconstruction of shear modulus of biological tissues from elastic internal displacement fields having in mind some application in quasi-static elastography.

More precisely, we are interested in a direct method, obtained from the reverse weak formulation of the linear elasticity equation, and where the elastic moduli are found by identifying the nullspace of a linear operator **A** constructed directly from the displacement fields.

The resulting method is efficient because it does not require iterative resolution of the forward problem nor additional information at the boundary. Another advantage is that it has stability properties under relatively weak regularity assumptions.

After presenting some stability results in a continuous viewpoint (see [1]), we will explain how to preserve its good properties after discretization of the operator A by a finite element approach.
In particular, we will show how to obtain quantitative error estimates (see [2]) for this inverse problem using a well-chosen pair of finite element spaces satisfying a generalized discrete inf-sup condition.

Numerical experiments will illustrate such stability results.

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MAPPING THE RELATIVE SHEAR MODULUS WITHIN BIOLOGICAL TISSUES FROM INTERNAL DISPLACEMENT FIELDS MEASURED IN QUASI-STATIC ULTRASOUND ELASTOGRAPHY

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Abstract

This presentation deals with relative shear modulus mapping within linear elastic and isotropic media, from the knowledge of displacement fields estimated in quasi-static ultrasound elastography (QSUE). QSUE mainly provides images of the internal displacements and strains that biological tissues experience under compression. Typically, the operator compresses the biological tissues with the probe while radiofrequency (RF) ultrasound images are acquired. With this technique, no boundary information is therefore available. Moreover, internal displacement fields are estimated from the acquired RF images, leading to 2D displacements, whereas the inverse problem is inherently a 3D one. In our recent work, mathematical developments for analyzing this inverse elasticity problem were introduced, as well as some initial results [1,2]. Here, to overcome the 2D/3D data problem, a plane stress-based approach is considered. The proposed method is based on the variational formulation of the equilibrium equations. This approach involves the use of test functions, chosen in this work to vanish at the domain boundary, because of the lack of information. Using a suitable finite element discretization technique leads to a finite dimensional homogeneous system of linear equations and the problem of reconstructing a relative shear modulus map is finally reduced to a null space identification problem of a large sparse matrix. With this method, only the displacement fields within the analyzed media need to be known beforehand. They are estimated using a method we previously developed for strain imaging [3].

Relative shear modulus mapping was assessed with plane stress and 3D numerical simulations, as well as data from phantom experiments (CIRS models 049 and 059). These different media consist of spherical inclusions embedded in homogeneous background materials. In all cases, the reconstructed maps allow a clear identification of the regions differing in stiffness, with inclusion-to-background modulus ratios (IBRs) that unambiguously reveal whether inclusions are stiffer or softer than the surrounding material. More specifically, for the plane stress simulations, resulting IBRs are very close to the theoretical values, whereas those obtained with the 3D simulations are, as expected, less accurate since the plane stress conditions are no longer satisfied. For the experimental data, a comparison with a clinical ultrasound scanner (Aixplorer, SuperSonic Imagine, France) was also conducted. Overall, for the phantom cases, the IBRs obtained with our approach are closer to the values given by the phantom manufacturer than the IBRs from the clinical system. Finally, results obtained with *in vivo* data of breast lesions will also be presented and discussed.

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RECENT ADVANCES IN CORRELATION-BASED SUPER-REVOLUTION FLUORESCNT MICROSCOPY: FROM SPARSE/NON-CONVEX TO GENERATIVE APPROACHES

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Abstract

Super-resolution fluorescence microscopy overcomes the physical barriers due to light diffraction, allowing for the observation of thin and small entities with sub-pixel precision. State-of-the-art super-resolution methods achieve adequate spatio-temporal resolution under rather challenging experimental conditions by means either of costly devices and/or very specific fluorescent molecules.

In this talk, we present a method for covariance-based super-resolution microscopy with intensity estimation and automatic parameter estimation which is well suited for live-cell imaging and which allows for improved spatio-temporal resolution by means of common microscopes and conventional fluorescent dyes. Our approach codifies the assumption of sparse distribution of the fluorescent molecules as well as the temporal and spatial independence between emitters in a covariance domain where the location of emitters is estimated by solving a non-convex optimisation problem where the ℓ_0 pseudo-norm is relaxed in an appropriate way. A separate intensity estimation step where intensity information is retrieved is then considered. This is a valuable piece of information for the biological interpretation of the results and their use in 3D super-resolution imaging, such as Multi Angle Total Internal Reflectance Fluorescence (MA-TIRF) microscopy. for which results are showed. To make the reconstruction model fully automated, we further detail automatic parameter selection strategies based on algorithmic restarting and on discrepancy-type approaches.

To conclude, we discuss how the latent fluctuation model can be estimated in a hybrid fashion combining sparsity constraints with Generative Adversarial Networks (GANs).

This is joint work with M. Cachia, V. Stergiopoulou, L. Blanc-Féraud (CNRS, I3S, Sophia-Antipolis), H. Goulart (IRIT, ENSEEIHT, Toulouse) and S. Schaub (IMEV, Villefranche-sur-Mer).

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A DEEP CONVOLUTIONAL NEURAL NETWORK FOR BRIGHTNESS ESTIMATION IN FLUORESCENCE IMAGING

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Abstract

Deep learning techniques have recently revolutionized the imaging field, often achieving impressive results and super-human level performance on complex computer vision tasks such as classification and object detection. Nonetheless, their use in biomedical imaging is often complicated by the limited amount of available, high-quality annotated data, which is crucial for training neural networks.

The successful application of a convolutional neural network (CNN) for brightness estimation of fluorescent spots in a point of care embedded system for viral diagnosis is presented [1]. For this problem, object detection is generally required to assess the presence of a molecule of interest, while brightness estimation is often performed to measure its concentration [2].

A new neural network for object detection and brightness estimation, which allows for completely automated fluorescence image analysis without requiring any input parameter from the user during the inference phase, is presented. Additionally, methods to assemble a good dataset with a limited number of images, as is typical with clinical data, will be discussed, together with strategies to reduce the computational load of the network through model optimization for deployment on resource-constrained embedded systems.

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SPATIALLY ADAPTIVE IMAGE RECONSTRUCTION IN ELECTRICAL IMPEDENCE TOMOGRAPHY

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Abstract

The inverse Electrical Impedance Tomography (EIT) problem involves collecting electrical measurements on the smooth boundary of a region to determine the spatially varying electrical conductivity distribution within the bounded region. Effective applications of EIT technology emerged in different areas of engineering, technology and applied sciences. The feasibility studies on potential medical imaging and biomaterial-engineering applications of EIT have focused on non-invasive approaches to monitoring and data reconstruction [1]. However, EIT is well known to suffer from a high-degree of nonlinearity and severe illposedness. Therefore, regularization is required to produce reasonable electrical impedance images. Using difference imaging, we propose a spatially-variant variational method which couples sparsity regularization and smoothness regularization for improved data reconstruction. The EIT variational model can benefit from structural prior information in the form of an edge detection map coming either from an auxiliary image of the same object being reconstructed or automatically detected.

We propose an efficient algorithm for minimizing the (non-convex) function based on Alternating Direction Method of Multipliers (ADMM). Preliminary experiments are presented which strongly indicate that using nonconvex versus convex variational EIT models holds the potential for more accurate reconstructions.

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HIGH SENSITIVITY IMAGING OF DEFECTS IN ELASTIC WAVEGUIDES USING NEAR RESONANCE FREQUENCIES

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Abstract

Reconstruction of defects in waveguides is of crucial interest in nondestructive evaluation of structures. This work is based on physical experiments done at Institut Langevin where one tries to reconstruct width defects in thin elastic plates using multi-frequency surface measurements (see [1]). Contrary to usual backscattering methods avoiding cut-off frequencies of the plate, they developed an experimental inversion method using these frequencies to obtain a high sensibility reconstruction of local defects in plates. We try here to provide a mathematical understanding of this inversion method and theoretical results on its stability.

Near resonance frequencies of the waveguide, the elastic problem is known to be ill-conditioned. Using the framework developed in [2], we prove that a phenomenon close to the tunnel effect in quantum mechanics can be observed and locally resonant modes propagate in the waveguide (see [3]). These modes are very sensitive to width variations, and measuring their amplitude enables reconstructing the local variations of the waveguide shape with very high sensibility. Given surface wavefield measurements for a range of near resonance frequencies, we provide a stable numerical reconstruction of the width of a slowly varying waveguide and illustrate it on defects like dilation or compression of a waveguide.

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RISING: A NEW FRAMEWORK FOR FEW-VIEW TOMOGRAPHIC IMAGE RECONSTRUCTION WITH DEEP LEARNING

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Abstract

In this talk we propose a new two-step procedure for sparse-view tomographic image reconstruction. It is called RISING, since it combines an early-stopped Rapid Iterative Solver with a subsequent Iteration Network-based Gaining step.

So far, regularized iterative methods have widely been used for X -ray computed tomography image reconstruction from low-sampled data, since they converge to a sparse solution in a suitable domain, as upheld by the Compressed Sensing theory. Unfortunately, their use is practically limited by their high computational cost which imposes to perform only a few iterations in the available time for clinical exams. Data-driven methods, using neural networks to post-process a coarse and noisy image obtained from geometrical algorithms, have been recently studied and appreciated for both their computational speed and accurate reconstructions. However, there is no evidence, neither theoretically nor numerically, that neural networks based algorithms solve the mathematical inverse problem modeling the tomographic reconstruction process.

In our two-step approach, the first phase executes very few iterations of a regularized model-based algorithm whereas the second step completes the missing iterations by means of a neural network. The resulting hybrid deep-variational framework preserves the convergence properties of the iterative method and, at the same time, it exploits the computational speed and flexibility of a data-driven approach.

Experiments performed on a simulated and a real data set confirm the numerical and visual accuracy of the reconstructed RISING images in short computational times.

A SCALED ADAPTIVE FISTA-LIKE ALGORITHM FOR SUPER-RESOLUTION IMAGE MICROSCOPY

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Abstract

In this talk we propose SAGE-FISTA [1], a Scaled Adaptive GEneralized FISTA-type algorithm for the minimization of the sum of two (possibly strongly) convex functions, one of which is assumed to be continuously differentiable. SAGE-FISTA can be considered as an accelerated version of the popular FISTA algorithm [2], where each iterate is obtained by first extrapolating the two previous iterates and then performing a proximal-gradient step at the extrapolated point. Unlike FISTA, the proposed algorithm employs an adaptive backtracking procedure, which allows for the non-monotone adjustment of the steplength along the iterations. This comes in handy when the initial guess for the steplength is extremely small, due to a coarse large estimate of the Lipschitz constant. The adaptive backtracking is further combined with a symmetric, positive definite and variable scaling matrix in the computation of the proximal-gradient step, by which some second-order information of the differentiable part of the objective function are incorporated while keeping the iteration computationally inexpensive, e.g. adopting Majorization-Minimization or Split-Gradient Techniques [3]. The algorithm also takes into account the inexact evaluation of the proximal operator and the possible introduction of the strong convexity moduli of the two functions into the inertial parameters.

If one or both functions are strongly convex, we prove that SAGE-FISTA converges linearly in the function values, with a convergence factor depending on both the strong convexity moduli of the two functions and the upper and lower bounds on the eigenvalues of the scaling matrices. Otherwise, we show that SAGE-FISTA retains the same O(1/k^2) convergence rate available for FISTA.

Furthermore, we apply our algorithm to an image super-resolution problem where a sparsity-promoting regularization function is coupled with a weighted least squares term [4]. Our numerical experiments show that SAGE-FISTA boosts the practical convergence speed with respect to standard implementations of FISTA, especially when used as inner solver of iteratively reweighted ℓ_1 schemes.

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A VARIATIONAL APPROACH FOR JOINT IMAGE RECOVERY-SEGMENTATION BASED ON SPATIALLY VARYING GENERALISED GAUSSIAN MODELS

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Abstract

The joint problem of reconstruction/feature extraction is a challenging task in image processing. It consists in performing, in a joint manner, the restoration of an image and the extraction of its features. The degradation usually corresponds to the application of a linear operator (e.g., blur, projection matrix) to the image, and the addition of a noise. Feature extraction problems arise when one wants to assign to the sought image a small set of parameters which can describe or identify the image itself. Image segmentation can be viewed as an example of features extraction, which consists of defining a label field on the image domain so that pixels are partitioned into a predefined number of homogeneous regions according to some specific characteristics.

In this work, we firstly propose a novel non-smooth and non-convex variational formulation of the joint problem of reconstruction/feature extraction. To do so, we consider the notion of *flexible sparse* regularization, introduced in [1]: for some $x = (x_i)_{1 \le i \le n} \in \mathbb{R}^n$, this amounts to considering penalties of the form $\sum_{i=1}^{n} |x_i|^{p_i}$, with positive exponents $(p_i)_{1 \le i \le n}$, for the variational regularisation of ill-posed inverse problems. To include this class of penalties in our formulation, we introduce a versatile prior that is based on a mixture of generalised Gaussian distributions (GGD) and whose parameters, including its exponents, are space-variant. We thus aim at jointly estimating an optimal configuration for $(p_i)_{1 \le i \le n}$, and retrieving the image. Under an assumption of consistency within the exponents values of a given region of the features space, we obtain the desired feature extraction starting from the estimated p_i parameters.

The specific non-convex structure of the resulting objective function, which is the sum of a coupling term and a set of block-separable terms, suggests the use of an alternating scheme: the idea is to sequentially update a subset of parameters through the resolution of an inner minimization problem, while the other parameters are assumed to be fixed. We design a new proximal alternating minimization algorithm that efficiently exploits the characteristic of the functions involved in our proposed variational formulation. We consider a hybrid scheme mixing both standard and linearised proximal steps on the different blocks of parameters. The novelty lies in the fact that the linearised step exploits a preconditioner for faster convergence and, contrarily to what usually happens in other methods belonging to this family, the linearisation involves a block-separable term, instead of the coupling term. We investigate the convergence properties for this algorithm based on the framework defined in [2].

Finally, we test the proposed approach in a joint deblurring/segmentation task for realistically simulated ultrasound images and compare its performance with state-of-the-art method in the field [3].

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OPTIMAL REGULARIZED ESTIMATION OF THE CROSS-POWER SPECTRUM FROM INDIRECT MEASUREMENTS: THEORETICAL RESULTS AND APPLICATION TO BRAIN CONNECTIVITY

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Abstract

We consider the problem of estimating the cross-power spectrum of a multivariate stochastic process $X(t)$ when only indirect measurement are available through a second observable stochastic process $Y(t)$. We assume the two processes to be related by a linear model of the form

$$Y(t) = GX(t) + N(t) \qquad (1)$$

A typical approach to this problem consists in two steps. First and estimate of $X(t)$ is obtained from the recorded data by solving the inverse problem described by equation (1). Then the cross-power spectrum of such an estimate is computed.

In [1] and [2] we have shown, analitically and through simulations, respectively, that if the inverse problem is solved via Tikhonov regularization, an unexpected parameter tuning issue may rise. Indeed the value of the regularization parameter that provides the best reconstruction of the unknown $X(t)$ (i.e. minimum ℓ_2-norm of the reconstruction error) is in general suboptimal for the subsequent estimation of the cross-power spectrum.

First, I will present the theoretical results we obtained by assuming both the signal $X(t)$ and the noise $N(t)$ to be white Gaussian noise. Then, I will show, through numerical simulations, that the analytical results hold also in case of more complex signals. Finally, I will indtroduce some preliminary results on the potentials of a one-step approach to the problem, i.e when the cross-power spectrum of $X(t)$ is directly estimated from that of $Y(t)$. At the end, I will discuss the implication of such results when studying functional brain connectivity from magneto-/electro-encephalographic data, i.e. when estimating the statistical relationship between the activity of spatially distinct brain areas from the electromagnetic field non invasively recorded outside the scalp [3, 4].

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FREE HYPER-PARAMETER SELECTION AND AVERAGING IN MAGNETO/ELECTRO-ENCEPHALOGRAPHY INVERSE PROBLEM

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Abstract

We present an innovative method for exploiting Sequential Monte Carlo (SMC) samplers structure in order to obtain free hyper-parameter selection and averaging for a large class of inverse problems where the likelihood depends on a scalar hyper-parameter for which an hyper-prior is available.

Indeed, considering a sequence of intermediate SMC samplers distributions such that each one can be interpreted as a posterior distribution conditioned on a different value for the hyper-parameter, we can exploit the natural approximation of the normalizing constant provided by the algorithm to obtain either selection or marginalization of the hyper-parameter.

The proposed approach is substantially different from the standard alternative where the hyper-parameter is itself sampled and estimated directly by the SMC samplers because particles from all iterations are effectively utilized in the final estimate, therefore avoiding the waste of computational time that is typical of SMC samplers.

In order to give an example for the construction of such sequences, we focus on a class of inverse problems where the likelihood belongs to the Natural Exponential Family (NEF), showing that in this context the most natural choice for the sequence of distributions - the one obtained growing the likelihood to an increasing exponent from zero to one - is suitable for the proposed method.

The most straightforward application is in the context of additive Gaussian noise inverse problems, where the noise standard deviation plays the role of the hyper-parameter, such as in the field of source localization from Magneto/Electro-Encephalography (MEG) data [1,2].

For analyzing the performance of the proposed method, we provide a comparison between the performances of the proposed method and the standard alternative for the resolution of both a toy example and the MEG inverse problem, showing that the proposed method provides better results with less computational cost.

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MINISYMPOSIUM
M26: Variational Methods for Inverse Problems in Imaging

Organizers:
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Variational methods nowadays rank among the most powerful and flexible approaches to tackle inverse problems in imaging. These include real-world applications like magnetic resonance imaging, positron emission tomography, computer tomography, transmission electron microscopy and many other recovery problems in medicine, engineering, and life sciences. Further, variational methods have proven themselves highly valuable for data pre-processing and image post-processing such as denoising, deblurring and inpainting. The recent years have seen several new developments in this area, coming from different mathematical backgrounds and being applicable for a variety of inverse problems. These developments include, for instance, deep learning approaches, tensorial lifting strategies, as well as novel optimal-transport- and total-variation-based regularizers. On the other hand, new techniques such as deep neural networks for inverse problems are inspired by variational methods. The aim of this minisymposium is to bring together experts with various backgrounds to discuss these recent achievements in the context of inverse problems in imaging, and to initiate potential new research directions and collaborations.

ROBUST PCA VIA REGULARIZED REAPER AND MATRIX-FREE PROXIMAL ALGORITHMS

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Abstract

The Principal Component Analysis (PCA) is a dimension reduction technique, where the dimension of a given data set is reduced by projecting them to a low-dimensional, affine subspace. Classically, the subspace which minimizes the sum of the squared Euclidean distances between the data points and their projections is chosen. This PCA approach is however very sensitive to outliers, so that various robust approaches were developed in the literature.

A recent robust model, called REAPER [1], aims to find the principal components by solving a convex optimization problem. Usually, the number of principal components must be determined in advance and the minimization is performed over symmetric positive semi-definite matrices having the size of the data, although the number of principal components is substantially smaller. This prohibits its use if the dimension of the data is large, which is often the case in image processing.

In this talk, we propose a regularized version of REAPER, called rREAPER [2], which enforces the sparsity of the number of principal components by penalizing the nuclear norm of the corresponding orthogonal projector. If only an upper bound on the number of principal components is available, our approach can be combined with the L-curve method to reconstruct the appropriate subspace. To solve the variational problem numerically, we introduce a matrix-free algorithm to find a minimizer of rREAPER which is also suited for high dimensional data. The algorithm couples a primal-dual minimization approach [3] with a thick-restarted Lanczos process [4]. This appears to be the first efficient convex variational method for robust PCA that can handle high-dimensional data. As a side result, we discuss the topic of the bias in robust PCA.

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JOINT EXIT WAVE RECONSTRUCTION AND IMAGE REGISTRATION AS A LEAST-SQUARES PROBLEM

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Abstract

Images generated by a transmission electron microscope (TEM) are blurred by aberrations from the objective lens and can be difficult to interpret correctly. One possible solution to this problem is to reconstruct the so-called exit wave, i.e. the electron wave in the microscope right before it passes the objective lens, from a series of TEM images acquired with varying focus. The forward model of simulating a TEM image from a given exit wave is known and easy to evaluate. In fact, it can be represented as weighted auto-correlation of the exit wave Ψ with the transmission cross-coefficient (TCC) T. More specifically, $\Psi *_T \Psi$ is the Fourier space representation of the corresponding TEM image. Here, the TCC encapsulates all assumptions on the electron microscope and the observed sample. However, it is in general not possible to reconstruct the exit wave from a series of images analytically. Here, it is important to note that the exit wave is complex-valued, while the TEM images are only real-valued. The corresponding inverse problem can be formulated as a minimization problem, which is done in the well known multiple input maximum a-posteriori (MIMAP) algorithm [1] and the maximum-likelihood (MAL) algorithm [2].

We propose a generalization of these methods by performing the exit wave reconstruction and the registration of the image series simultaneously using the functional

$$E : L^2(A, C) \times (R^2)^N \to R,$$

$$(\Psi, t) \mapsto \frac{1}{N} \sum_{j=1}^{N} \left\| \Psi *_{T_{Z_j}} \Psi - F(g_j \circ \phi_{t_j}) \right\|_{L^2}^2 + \alpha \left\| \Psi - \Psi_M \right\|_{L^2}^2 .$$

Here, $g_1, \ldots, g_N$ are TEM images corresponding to focus values $Z_1, \ldots, Z_N$, ϕ_{t_j} is a translation by the vector t_j and Ψ_M is a fixed a-priori estimate of the exit wave. We show that the objective functional is not convex with respect to the exit wave for $\alpha > 0$ sufficently small, which also carries over to the MAL and MIMAP functionals.

The main result is the existence of minimizers of the objective functional. These results are based on the properties the weighted cross-correlation and certain assumptions on the TCC. A key ingredient here is the uniform approximation of the TCC with a sum of $*$-seprable weights.

Finally, the practical applicability of our method is verified with numerical experiments on simulated and real input data.

Note that part of this work has been published in [3].

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OPTIMAL-TRANSPORT-BASED APPROACHES FOR DYNAMIC IMAGE RECONSTRUCTION

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Abstract

We discuss the solution of dynamic inverse problems in which for each time point, a time-dependent linear forward operator mapping the space of measures to a time-dependent Hilbert space has to be inverted. These problems are regularized with dynamic optimal-transport energies that base on the continuity equation as well as convex functionals of Benamou-Brenier-type. Well-posedness of respective Tikhonov minimization is discussed in detail. Further, for the purpose of deriving properties of the solutions as well as numerical algorithms, we present sparsity results for general inverse problems that are connected with the extremal points of the Benamou-Brenier energy subject to the continuity equation. For the latter, it is proven that the extremal points are realized by point masses moving along curves with Sobolev regularity. This result will be employed in numerical optimization algorithms of generalized conditional gradient type. We present instances of this algorithm that are tailored towards dynamic inverse problems associated with point tracking. Finally, the application and numerical performance of the method is demonstrated for sparse dynamic superresolution.

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DEEP NEURAL NETWORKS FOR INVERSE PROBLEMS WITH PSEUDODIFFERENTIAL OPERATORS: AN APPLICATION TO LIMITED ANGLE TOMOGRAPHY

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Abstract

Computed Tomography makes use of computer-processed combinations of many X-ray measurements of an object, taken from different angles, and attempts to recover the inner structure of the object from the data. In the case of limited-angle tomography, the reconstruction problem is severely ill-posed and the traditional reconstruction methods, e.g. filtered backprojection (FBP), do not perform well. In this work, we investigate a brand-new method for limited-angle tomography reconstruction, based on the unrolled version of the ISTA algorithm where each iteration contains a convolutional neural network (CNN). The idea of this project has emerged from the observation that the backprojection operator can be approximated by a sequence of convolutions applied to the original object in the wavelet domain. Thus, each CNN has the same structure as the previously mentioned sequence of convolutions. The preliminary results on 128x128 images, highly encouraging, show that our model can provide reconstructions that are much more accurate than the reconstructions provided by the standard ISTA algorithm.

TOWARDS OFF-THE-GRID ALGORITHMS FOR TOTAL VARIATION REGULARIZED INVERSE PROBLEMS

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Abstract

The choice of a regularizer has a crucial impact on the structure of the solutions of an inverse problem. It is now well understood which regularizations tend to promote sparse vectors, low rank matrices, or piecewise constant images. Yet, such structural properties are rarely exploited in optimization algorithms. In most cases, they only appear in the limit, when the algorithm has converged.

In this work, we focus on the total (gradient) variation which has been widely used in imaging following the pioneering work of Rudin, Osher and Fatemi [1]. Such a regularization is used to recover piecewise constant ("cartoon") solutions. Indeed, it is known that some solutions are the sum of a few indicator functions [2,3]. Inspired by recent works on the recovery of point sources [4], we propose to exploit that structure in a Frank-Wolfe based algorithm which progressively adds shapes which are the solution a variational geometric problem which is similar to the Cheeger problem.

The benefit of this approach is that the reconstructed signal does rely on a computation grid, therefore it does not suffer from traditional reconstruction artifacts such as anisotropy or blur. A preprint is available at [5].

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PIXEL-DRIVEN TOMOGRAPHY: ANALYSIS AND THE GRATOPY TOOLBOX

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Abstract

The Radon transform is a cornerstone of countless imaging applications associated with tomography and is therefore often considered in the context of inverse problems [1]. The use of computers in tomographic reconstruction requires discretization of the Radon transform and its adjoint – the backprojection. However, the design of discretizations is neither trivial nor obvious. Many applications use different discretization approaches for the Radon transform and the backprojection, which can have a negative impact on the convergence of iterative solvers. Moreover, using discretizations of operators – and not the infinite-dimensional operator – in the context of variational methods for inverse problem can be understood as the usage of an inexact operator. A complete analysis of a variational approach has to take such operator errors into account; cf. [2]. Hence, one needs to know this error – in the best case in a uniform sense – but often, this is ignored. A well-known discretization approach to the Radon transform – and more general projection methods – is the 'Pixel-Driven' method [3], which is typically used for the backprojection but anecdotally known to be a poor approximation for the forward operator, showing distinct oscillation artifacts [4]. The word 'anecdotal' reveals the fact that there is very little rigorous mathematical analysis of such methods. We present such an analysis for Pixel-Driven methods, showing convergence in the operator norm towards the Radon transform between L^2 spaces when the discretization parameters are chosen appropriately [5]. Note that while with such parameters, convergence – including rates – can be obtained, this suitable parameter choice does not coincide with the standard choice, thus explaining the anecdotal poor approximation quality. We further present the Gratopy toolbox [6], a Python-OpenCL-based toolbox allowing for highly efficient, versatile, and easy application of Pixel-Driven methods to make use of said theoretical results.

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STEKLOV AND MODIFIED TRANSMISSION EIGENVALUES AS TARGET SIGNATURES IN AN INVERSE FLUID-SOLID INTERACTION PROBLEM

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Abstract

We revisit total variation denoising and study an augmented model where we assume that an estimate of the image gradient is available. We show that this increases the image reconstruction quality and derive that the resulting model resembles the total generalized variation denoising method, thus providing a new motivation for this model. Further, we propose to use a constraint denoising model and develop a variational denoising model that is basically parameter free, i.e., all model parameters are estimated directly from the noisy image. Moreover, we use Chambolle–Pock's primal dual method as well as the Douglas–Rachford method for the new models. For the latter one has to solve large discretizations of partial differential equations. We propose to do this in an inexact manner using the preconditioned conjugate gradients method and derive preconditioners for this. Numerical experiments show that the resulting method has good denoising properties and also that preconditioning does increase convergence speed significantly.

Finally, we analyze the duality gap of different formulations of the TGV denoising problem and derive a simple stopping criterion.

This is joint work with M. Levitin (University of Reading) and P. Monk (University of Delaware).

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PENALIZED METHODS FOR SOLVING CONSTRAINED VARIATIONAL PROBLEMS IN IMAGE RECOVERY

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Abstract

Many image recovery problems can be efficiently addressed by minimizing a smooth cost function subject to various constraints. In this talk, we present a new method for solving large-scale constrained differentiable optimization problems. To account efficiently for a wide range of constraints, our approach embeds a subspace algorithm in an exterior penalty framework. The subspace strategy, combined with a Majoration-Minimization step search, takes great advantage of the smoothness of the penalized cost function. Assuming that the latter is convex, the convergence of our algorithm to a solution of the constrained optimization problem is proved.

Numerical experiments carried out on a large-scale image restoration application show that the proposed method outperforms state-of-the-art algorithms in terms of computational time.

VARIATIONAL METHODS FOR FUNCTIONAL AND QUANTITATIVE MRI

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Abstract

Inverse problems are an essential part of functional and quantitative procedures in MRI. These MRI methods typically require multiple measurements of specific regions for the intended analysis. In this context, variational reconstruction techniques can contribute considerably to an increased spatial-temporal resolution with integrated denoising and sub-sampling artifact suppression. The determination of biophysical parameters is typically a separate inverse problem, but can also be integrated into a model-based reconstruction where the physical model connects the different sample volumes. Here we show how special problem-adapted regularization functions enable significant progress in dynamic MR imaging, brain perfusion determination by ALS and for the determination of biophysical parameters.

An essential aspect of regularization for functional and quantitative methods is the combination of regularization terms for individual components of the information base. Thus, with the infimal convolution of total generalized variation (ICTGV) a flexible regularization can be achieved, which spatially models fast and slow signal changes [1]. For arterial spin labeling (ASL), two different image data sets have to be measured and combined, for which a combined regularization model has proven to be particularly suitable [2, 3]. Simulated data were used to optimize regularization ("data driven learning"). It was observed that the parameter sets found were quite stable for realistic data variations. For parameter quantification, both in model-based reconstruction and in image space, the regularization functionals are joined using a Frobenius norm to exploit shared features between the parameters of interest. Jointly regularizing preserves fine structures which could be otherwise wrongly classified as noise [4, 5].

The impact and potential for improvements of these regularization strategies together with parallel imaging and tailored sub-sampling strategies will be shown for cutting edge problems. Furthermore the advantages and challenges compared to the recently implemented deep learning strategies will be discussed.

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ACCELERATING THE SOLUTION OF SPARSE DYNAMIC INVERSE PROBLEMS USING TOOLS FROM DYNAMIC PROGRAMMING

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Abstract

Recent work has shown that sparse dynamical inverse problems, such as particle tracking, can be reformulated into a convex variational optimization problem over the space of measures. The resulting problem resembles a weighted Lasso problem for sparsity combined with an optimal transport regularization to enforce regularity in time. Early numerical results using a Frank-Wolfe algorithm are promising, we focus on improving efficiency for larger scaled problems.

Frank-Wolfe is a greedy method which at each iteration approximates the best single path to fit the remaining data error. This sub-problem is the main computational challenge as it is very high dimensional and non-convex. We observe that discretizing this 'best path' problem allows it to be reformulated as a classical shortest path problem on a directed acyclic graph. Leveraging this observation enables us to produce much more efficient algorithms, either using uniform or random discretizations of the sub-problem.

Our approach is validated analytically and numerically. We first prove that our stochastic Frank-Wolfe algorithm converges to the globally optimal energy almost-surely. The numerical reconstructions are much faster than previous methods and we compare the performance of random and uniform discretizations.

MINISYMPOSIUM
M27: Inverse Problems in Scattering Theory and Geometry

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Inverse scattering problems and geometrical inverse problems are classical and active subfields of inverse problems. The aim in inverse scattering theory is to recover information about some unknown medium or potential function from measurements conducted far away. Correspondingly, often in geometrical inverse problems the measurement is conducted at the boundary of a region of interest. Both fields have recently seen many breakthroughs, especially in the non-linear setting.

This mini-symposium aims to collect both experts and newcomers from different fields studying scattering theory and geometrical inverse problems. Both theoretical and numerical sides of the problems are welcome.

DETERMINING A LORENTZIAN METRIC FROM THE SOURCE-TO-SOLUTION MAP FOR THE RELATIVISTIC BOLTZMANN EQUATION

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Abstract

In this talk, we consider the following problem: Given the source-to-solution map for a relativistic Boltzmann equation on a neighbourhood V of an observer in a Lorentzian spacetime (M,g) and knowledge of $g|_V$, can we determine (up to diffeomorphism) the spacetime metric g on the domain of causal influence for the set V?

We will show that the answer is yes. In this talk, we will review results and techniques developed in the study of inverse problems similar to ours. We will also introduce the relativistic Boltzmann equation and comment on existence of solutions to this PDE given some initial data. We will sketch the key ideas of the proof of our result. One such key point is that the nonlinear term in the relativistic Boltzmann equation which describes the behaviour of particle collisions captures information about a source-to-solution map for a related linearized problem. We use this relationship together with an analysis of the behaviour of particle collisions by classical microlocal techniques to determine the set of locations in V where we first receive light particle signals from collisions in the unknown domain. From this data we are able to parametrize the unknown region and determine the metric.

The new results presented in this talk are joint work with Antti Kujanapää, Matti Lassas, and Tony Liimatainen.

STABILITY OF GEL'FAND'S INVERSE BOUNDARY SPECTRAL PROBLEM VIA THE UNIQUE CONTINUATION

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Abstract

Gel'fand's inverse boundary spectral problem concerns determining the geometry of a compact manifold with boundary from the boundary spectral data for the Laplacian. We show that this problem has a stable solution with quantitative stability estimates in a class of Riemannian manifolds with bounded geometry. More precisely, we show that finitely many Neumann eigenvalues and the boundary values of the corresponding eigenfunctions, known up to small errors, determine a metric space that is close to the manifold in the Gromov-Hausdorff topology. This result is based on an explicit estimate on the stability of the unique continuation for the wave operator from a subset of the manifold boundary.

References

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THE FIXED ANGLE SCATTERING PROBLEM WITH A FIRST-ORDER PERTURBATION

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Abstract

In this talk, we consider a fixed angle inverse scattering problem associated with the magnetic Schrödinger operator in dimensions greater or equal than 2. We prove that the magnetic field and the electric potential are uniquely determined by scattering measurements corresponding to finitely many measurements. We also show that the number of measurements can be reduced if the coefficients have certain symmetries.

The talk will be based on joint work [1] with Mikko Salo (University of Jyväskylä, Finland) and Cristóbal Meroño (Universidad Politécnica de Madrid, Spain).

References

1. C. J. Meroño, L. Potenciano-Machado and M. Salo, The fixed angle scattering problem for the Schrödinger operator with a first-order perturbation, Ann. Henri Poincaré, 22, 3699-3746 (2021).

SIMULTANEOUS RECOSTRUCTION OF CONDUCTIVITY, BOUNDARY SHAPE AND CONTACT IMPEDANCES IN EIT

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Abstract

The objective of electrical impedance tomography is to reconstruct the internal conductivity of a physical body based on current and voltage measurements at the boundary of the body. In many medical applications the exact shape of the domain boundary and contact impedances are not available. This is problematic as even small errors in the boundary shape of the computation domain or in the contact impedance values can produce large artifacts in the reconstructed images, which results in a loss of relevant information. A method is proposed that simultaneously reconstructs the conductivity, the contact impedances, and the boundary shape from EIT data. The approach consists of three steps: first, the unknown contact impedances and an anisotropic conductivity reproducing the measured EIT data in a model domain are computed. Second, using isothermal coordinates, a deformation is constructed that makes the conductivity isotropic. The final step minimizes the error of true and reconstructed known geometric properties (like the electrode lengths) using conformal deformations.

The feasibility of the method is illustrated with experimental EIT data, with robust and accurate reconstructions of both conductivity and boundary shape.

This is joint work with Juan Pablo Agnelli, Ville Kolehmainen, Matti Lassas and Samuli Siltanen.

INVERSE SCATTERING PROBLEMS FOR A BIHARMONIC OPERATOR

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Abstract

In this talk we will discuss inverse scattering theory for a first-order perturbation of a biharmonic operator. The coefficients of the biharmonic operator are vector- and scalar-valued and they can be complex and singular. We show that the direct scattering problem is well-posed in certain weighted Sobolev spaces under suitable radiation conditions.

For the inverse problem, the measurement is the scattering amplitude, obtained from the asymptotics of the scattering solutions at infinity. We show that in dimensions higher than one it is possible to uniquely recover a combination of the potential functions given the scattering amplitude at all angles at high wave numbers. For the problem of recovering the potentials from measurements of backscattering, we show the jumps and singularities of the potentials can be found in dimensions one, two and three. We will also discuss possible numerical approaches to the problem.

This is a joint work with Valery Serov, Markus Harju and Jaakko Kultima (University of Oulu).

MINISYMPOSIUM

M28: Regularization Methods and Applications in Statistics and Econometrics

Organizers:
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Ill-posed inverse problems arise in many applications of statistics and econometrics. Typical examples are the estimation of a density function (with the deconvolution problem) or of a regression function (with the nonparametric instrumental regression). One main difference with standard cases is that the operator characterizing the inverse problem is a statistical object that is often unknown and must be estimated.

The objective of this mini-symposium is to gather experts and young researches to discuss about recent advances in regularization methods and applications in statistical and econometrics issues.

TOTAL VARIATION-BASED LAVRENTIEV REGULARISATION OF MONOTONE PROBLEMS

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Abstract

We study total variation-based (nonlinear) Lavrentiev regularisation of monotone ill-posed problems

$$A(u) = f\,. \tag{1}$$

Inspired by the efficient taut string-method [1–4] for sparse denoising of piecewise constant spline-signals in which A is the identity, we propose algorithms for total-variation denoising with monotone operators A in Volterra integral equations of the first kind and certain parameter-identification problems.

The new regularisation method is shown to be well-posed for quite general monotone operators and regularisation functionals in Banach spaces, and we also provide some convergence-rate results.

References

1. P. L. Davies and A. Kovac, *Local Extemes, Runs, Strings and Multiresolution*, The Annals of Statistics, **29**(1), 1–65, (2001).
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LOCALIZATION OF MOVING SOURCES: UNIQUENESS, STABILITY, AND BAYESIAN INFERENCE

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Abstract

We consider the scalar wave equation in R^3 with a moving point source,

$$c^{-2}\partial_t^{\,2}u(t,x) - \Delta u(t,x) = q(t)\delta(x - p(t)),$$

where $q \in C^m((0,T))$, $q \geq 0$, $p \in C^{m+1}((0,T),\Omega)$, $\Omega \subset R^3$ is open, bounded and simply connected, and with vanishing initial conditions. For the direct problem we prove that $u(t,\cdot)$ is in $C^\infty((0,T), H^{m-1/2-\varepsilon}(R^3))$ for any positive ε. We also prove uniqueness and stability results for the associated inverse point source problem in the case the wave field measurement boundary $\partial\Omega$ is real-analytic.

Next, we present and investigate numerically a Bayesian framework for the inference of the source trajectory and intensity from wave field measurements. The framework employs Gaussian process priors, the pre-conditioned Crank-Nicholson scheme with Markov Chain Monte Carlo sampling, and conditioning on functionals to include prior information on the source trajectory.

References

1. G. Hu, Y. Kian, P. Li, Y. Zhao, Inverse moving source problems in electrodynamics, *Inverse Problems*, 35, 075001 (2019).
2. T. Ohe, Real-time reconstruction of moving point/dipole wave sources from boundary measurements, *Inverse Problems in Science and Engineering*, 28 (2020).

ON THE DECONVOLUTION OF RANDOM VARIABLES

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Abstract

We use mollification to regularize the problem of deconvolution of random variables. This regularization method provides a unifying framework for comparing the advantages of various filter-like techniques such as deconvolution kernels, Tikhonov or spectral cutoff. In particular, the mollification approach allows to relax some restrictive assumptions required by deconvolution kernels, and has better stabilizing properties compared to spectral cutoff or Tikhonov. We show that this approach achieves optimal convergence rates on the Besov and Sobolev classes. The qualification can be arbitrarily high depending on the choice of approximate unit used. We propose an adaptive choice of the regularization parameter using the Lepskii method and we will illustrate our point by means of numerical simulations.

References

1. T. Hohage, P. Maréchal, L. Simar and A. Vanhems, A mollifier approach to the deconvolution of probability densities, Working paper, https://dial.uclouvain.be/pr/boreal/object/boreal:259425

SUPERMIX: SPARSE REGULARIZATION FOR MIXTURES

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Abstract

This contribution investigates the statistical estimation of a discrete mixing measure μ^0 involved in a kernel mixture model. Using some recent advances in ℓ_1-regularization over the space of measures, we introduce a "*data fitting +regularization*" convex program for estimating μ^0 in a grid-less manner, this method is referred to as Beurling-LASSO.

Our contribution is two-fold: we derive a lower bound on the bandwidth of our data _tting term depending only on the support of μ^0 and its so-called \minimum separation" to ensure quantitative support localization error bounds; and under a so-called \non-degenerate source condition" we derive a non-asymptotic support stability property. This latter shows that for sufficiently large sample size n, our estimator has exactly as many weighted Dirac masses as the target μ^0, converging in amplitude and localization towards the true ones. The statistical performances of this estimator are investigated designing a so-called "*dual certificate*", which will be appropriate to our setting. Some classical situations, as e.g. Gaussian or ordinary smooth mixtures (e.g. Laplace distributions), will be discussed.

We stress in particular that our method is completely adaptive w.r.t. the number of components involved in the mixture.

ON HAMILTON-JACOBI PDES AND IMAGE DENOISING MODELS WITH CERTAIN NON-ADDITIVE NOISE

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Abstract

We consider image denoising problems formulated as variational problems. It is known that Hamilton-Jacobi PDEs govern the solution of such optimization problems when the noise model is additive. In this work, we address certain non-additive noise models and show that they are also related to Hamilton-Jacobi PDEs. These findings allow us to establish new connections between additive and non-additive noise imaging models. With these connections, some non-convex models for non-additive noise can be solved by applying convex optimization algorithms to the equivalent convex models for additive noise.

Several numerical results are provided for denoising problems with Poisson noise or multiplicative noise.

This is joint work with Jerome Darbon and Tingwei Meng.

REGULARISATION, OPTIMISATION, SUBREGULARITY

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Abstract

Regularisation theory in Banach spaces, and non-norm-squared regularisation even in finite dimensions, generally relies upon Bregman divergences to replace norm convergence. This is comparable to the extension of first-order optimisation methods to Banach spaces. Bregman divergences can, however, be somewhat suboptimal in terms of descriptiveness. Using the concept of (strong) metric subregularity, previously used to prove the fast local convergence of optimisation methods, we show norm convergence in Banach spaces and for non-norm-squared regularisation. For problems such as total variation regularised image reconstruction, the metric subregularity reduces to a geometric condition on the ground truth: flat areas in the ground truth have to compensate for the fidelity term not having second-order growth within the kernel of the forward operator. Our approach to proving such regularisation results is based on optimisation formulations of inverse problems.

As a side result of the regularisation theory that we develop, we provide regularisation complexity results for optimization methods: how many steps $N\delta$ of the algorithm do we have to take for the approximate solutions to converge as the corruption level $\delta \searrow 0$?

References

1. D. Burago, S. Ivanov, M. Lassas, J. Lu, Stability of the Gel'fand inverse boundary problem via T. Valkonen, Regularisation, optimization, subregularity, *Inverse Problems*, 37, 045010(2021).

A MOLLIFIER APPROACH TO THE NONPARAMETRIC INSTRUMENTAL REGRESSION PROBLEM

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Abstract

We propose a mollification method for the problem of instrumental regression. We show that our estimator is consistent, and compare its performances to that of the classical regularization methods. A finite sample study enables us to demonstrate the efficiency of mollification compared to other estimation methods.

References

1. Xavier Bonnefond and Pierre Marechal. A variational approach to the inversion of some compact operators. *Pacific journal of optimization*, 5(1) (2009).
2. Marine Carrasco, Jean-Pierre Florens, and Eric Renault. Linear inverse problems in structural econometrics estimation based on spectral decomposition and regularization. *Handbook of econometrics*, 6, (2007).

Poster Session

STABILITY FOR A SPECIAL CLASS OF ANISOTROPIC CONDUCTIVITIES VIA AN AD-HOC MISFIT FUNCTIONAL

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Abstract

In recent years there has been an increasing interest in the Calderòn problem with anisotropic conductivity. In particular, the geometric nature of the problem has been extensively studied to determine when uniqueness and stability occur. In our latest work, we have followed a different line of research, inspired by the ideas introduced by Alessandrini and Vessella in [1], where they proved Lipschitz stability for piecewise constant isotropic conductivities. We have managed to extend their argument to a special class of anisotropic conductivities.

In this poster we are going to review the stability estimate in terms of a novel misfit functional and then derive as a corollary a Lipschitz stability estimate in terms of the classical local Dirichlet-to-Neumann map.

References

1. G. Alessandrini, S. Vessella, Lipschitz stability for the inverse conductivity problem, *Adv. Appl. Math.* **35**, 207-241 (2005).

JACOBIAN OF SOLUTIONS TO THE CONDUCTIVITY EQUATION IN LIMITED VIEW

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Abstract

We address the Jacobian of two solutions to the conductivity equation in a limited view setting for a domain in two-dimensional Euclidean space. The limited view setting is characterized by a part of the boundary that we can control with a Dirichlet boundary condition and a part of the boundary, where the solution is zero. Our work is motivated by the Radó-Kneser-Choquet theorem which states that under certain conditions on the boundary functions the Jacobian of the solutions is non-vanishing. This result has been generalized from the case of a constant isotropic conductivity to anisotropic conductivities. The key novelty of the presented work is the generalization of this theorem from the full view to the limited view setting for anisotropic conductivities. We show under which condition on the boundary function the corresponding solution has no critical points. Moreover, how this result can be extended so that the Jacobian of two solutions is non-vanishing. This is related to the number of arcs on the boundary along which the boundary functions are non-increasing and non-decreasing.

This type of result is used in inverse problems for reconstructing the conductivity from interior power densities. Here the non-vanishing Jacobian condition ensures invertability of the matrix composed of power densities.

We illustrate how this result can be used for reconstruction of an isotropic conductivity from power densities by a numerical example.

OPTIMAL EXPERIMENTAL DESIGN FOR SOUND SOURCE LOCALIZATION

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Abstract

Acoustic source localization has a variety of application areas. Especially, methods using the inverse scheme with processed measurement data captured by microphone arrays have been commonly used over the last decade.

In this poster, we propose a method for optimal experimental design for the acoustic source identification problem. Optimizing the design of experiments is important so that the uncertainty in the estimated parameters is minimized. The source that needs to be identified is sparse and infinite-dimensional, which is challenging from both mathematical and computational points of view. Our approach relies on an iteratively reweighted least-squares scheme.

This is a joint work with Barbara Kaltenbacher, Daniel Walter and Konstantin Pieper.